HEWLETT hp PACKARD

NOISE FIGURE PRIMER

application note 57

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INTRODUCTION

The ultimate sensitivity of a detection system is set by the noise presented to the system with the signal. In addition, any detection system adds additional noise in its detection and amplification process. Since the noise contribution of the detection system is usually the larger of these two, and since the level of input noise is generally beyond our control, our approach is to study, measure, and attempt to minimize the noise contribution of the detection system.

BASIC CONSIDERATIONS

A microwave input termination has a certain amount of available noise power which it can deliver to a matched system. For definitions, let's first consider lower frequencies where we can work with lumped constants.

A resistance R at temperature T generates across its open-circuited terminals a voltage resulting from the random motion of free electrons, thermally agitated. This "noise voltage" e_n is infinitely broadbanded and can be defined by the equation:

$$e_n{}^2/B = 4kTR \text{ (volts)}^2\text{/unit frequency bandwidth}$$

where

k = Boltzmann's constant, 1.374×10^{-23} joule/^{o}K

T = absolute temperature, ^{o}K

R = resistance (or resistive component of impedance)

B = bandwidth

Generally our noise considerations will be concerned with a finite bandwidth, and we may use the more familiar notation of the equation.

$$e_n{}^2 = 4kTRB \qquad (1)$$

If the resistance R is connected to a matched load resistance (R_L = R), maximum transfer of the noise power will occur. Noise power P_n dissipated in the load resistance R_L due to the noise voltage generated in the original resistance R will have the value

$$P_n = kTB \qquad (2)$$

Equation (2) defines the available noise power from the original resistance. The actual noise power dissipated in the load resistance could be affected by any loss in the connecting leads, a less than perfect match to the original resistance and the noise power generated within the load resistance itself.

In systems operating at frequencies where voltages and resistances cannot be clearly defined, equation (2) becomes the usable expression, containing terms that can be measured.

In certain low noise applications, a deliberate (and carefully determined) mismatch is created between the input termination and the detection device. This technique couples less than the available termination noise power (kTB) into the detection device. However, this system consideration is beyond the scope of this article.

In a microwave receiver, the "input termination" is an antenna coupled to the atmosphere; in an IF strip, the input termination is generally a mixer of some sort. In either case, the termination has an available noise power given by P_n = kTB. If there were a perfect amplifier or receiver which added no noise in the amplification process (and if it were perfectly matched to its input termination) its output noise power would be kTBG, where G is the power gain of the system.

A figure of merit for an actual microwave receiver or IF amplifier is the ratio of actual output noise power when T = 290^{o} to the theoretical minimum, Noise Figure, F, is this figure of merit referred to room temperature. (T = 290^{o}K.)

Thus

$$F = \frac{N_1}{kT_oBG} \qquad (3)$$

where N_1 is the measured noise power output of the system when T_o = 290^{o}. The perfect amplifier would of course have a noise figure of 1, (0 db).

Of the total noise power output of a system (N_1 = kT_oBGF) we know that a specific portion is the result of amplified input noise (kT_oBG). The amount of noise power added by the receiver (N_r) is the difference, or

$$\begin{aligned} N_r &= N_1 - kT_oBG \\ &= (F - 1)\, kT_oBG \end{aligned} \qquad (4)$$

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NOISE FIGURE MEASUREMENTS WITH A SIGNAL GENERATOR

Noise Figure ($F = N_1/kT_oBG$) can be calculated from measurements taken by a stable, well-attenuated signal generator and suitable power meter. We can measure T, which is the temperature of the input termination in °K; N_1 can be measured with a power meter at the system output.

To determine accurately the gain bandwidth product (BG), it is necessary to plot the response of the system and graphically integrate the curve...a time-consuming process. Approximations can be made on the basis of the 3 db bandwidth, which shortens the process. After measuring the 3 db points, there are graphs that may be entered to get an approximation of the effective noise bandwidth. Gain, of course, may be easily measured with a proper signal generator and power meter.

From this data, noise figure may be calculated. Often, however, this is only a first step, because the primary objective is to minimize noise figure by repositioning components, by substituting crystals or tubes, or tuning filter networks. Clearly, the many-step process represented by the Signal Generator-Power Meter method has limited value in such a situation, since a new measurement must be made at each readjustment.

THE NOISE SOURCE AS A BROADBAND SIGNAL GENERATOR

If a known level of broadband noise can be introduced at the input of a device under test, a differential power measurement at the output would indicate a gain bandwidth product of the device.

A gas discharge noise source operates as an input termination at a very high temperature, and has an available noise power much higher than the normal termination. The effective thermal agitation of an argon tube noise source, for example, represents an equivalent temperature of approximately 10,000 K° compared with room temperature of 290°K.

Available excess power from the fired noise source can be expressed in the same terms as the input termination,

$$P_{ns} = k(T_2 - T_o)B$$

where T_2 is the equivalent absolute temperature of the noise source.

Since a measurement of the device output both with and without the additional noise power input will give an indication of gain-bandwidth product, it is possible to compute noise figure with no further measurements, since all independent variables of equation (3) are known.

Using the system of figure 1, consisting of an input termination, an excess noise source, a receiver under test and an output power detector, it is possible to measure N_1 with the excess noise source "cold" and to measure N_2 with the excess source fired. N_1 is graphically illustrated in figure 2(a), and consists of the amplified input termination noise plus the noise generated within the receiver. N_2, illustrated in figure 2(b), consists of N_1 plus the amplified excess noise power viewed at the receiver output.

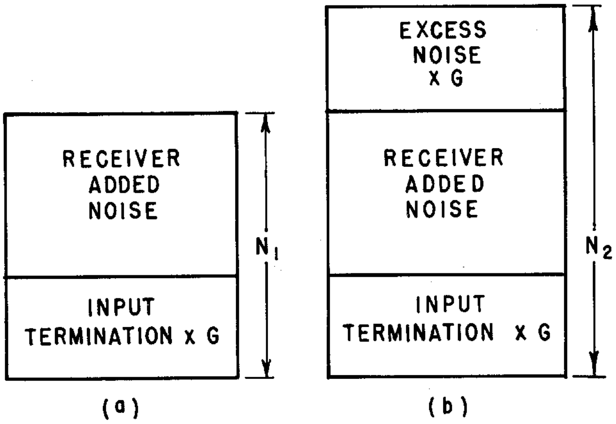

Figure 2. Representation of total noise power output of the system in figure 1 when the Excess Noise Source is "cold" (N_1), and when the Excess Noise Source is "fired" (N_2).

Taking the ratio of these measured powers we have:

$$\frac{N_2}{N_1} = \frac{\left(\begin{matrix}\text{input}\\ \text{termination}\end{matrix}\right) \text{x } G + (\text{receiver}) + \left(\begin{matrix}\text{excess}\\ \text{noise}\end{matrix}\right) \text{x } G}{\left(\begin{matrix}\text{input}\\ \text{termination}\end{matrix}\right) \text{x } G + (\text{receiver})}$$

Substituting from previous equations, and assuming measurement conditions of $T = T_o = 290°K$,

$$\frac{N_2}{N_1} = \frac{kT_oBG + (F-1)kT_oBG + k(T_2-T_o)\,BG}{kT_oBG + (F-1)kT_oBG}$$

$$= \frac{FT_o + (T_2-T_o)}{FT_o}$$

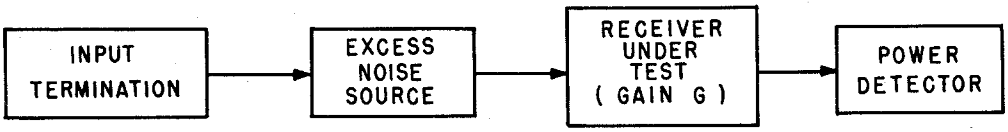

Figure 1. A typical noise figure measuring system.

and, finally

$$F = \frac{(T_2 - T_o)}{T_o} \times \frac{1}{\left(\frac{N_2}{N_1} - 1\right)}$$

Converting to logarithmic notation,

$$F_{db} = 10 \log \frac{(T_2 - T_o)}{T_o} - 10 \log\left(\frac{N_2}{N_1} - 1\right) \quad (5)$$

In equation (5) the ratio $(T_2 - T_o)/T_o$ is a measure of the relative excess noise power available from a noise source and is specified by the manufacturer. In the case of argon gas tubes, this ratio is 33.1; 10 log $(T_2 - T_o)/T_o$ is 15.2 db. When using such a tube equation (5) simplifies to,

$$F_{db} = 15.2 - 10 \log (N_2/N_1 - 1)$$

NOISE FIGURE MEASUREMENT WITH AN EXCESS NOISE SOURCE

Equation (5) opens the door to several measurement techniques utilizing the excess noise source. We shall consider the "twice-power" and "Y-Factor" manual techniques and an automatic approach to noise figure measurements.

A. Twice-Power Method of Manual Noise Figure Measurement

In actually measuring the "N_1" and "N_2" of equation (5), if N_2 was set to be twice N_1 then equation (5) reduces to:

$$F_{db} = 10 \log \frac{(T_2 - T_o)}{T_o} - 10 \log (1)$$

$$= 10 \log (T_2 - T_o)/T_o$$

With the proper equipment, the condition of $N_2 = 2\ N_1$ can be established by varying the relative excess noise power of the noise source. With the equipment of figure 3, the procedure would be:

1) Set a convenient reference on the power detector with the excess noise source "cold" and the 3 db pad out. This is N_1.

2) Insert the 3 db pad and fire the excess noise source.

3) Vary the rotary vane attenuator until the original power detector reference point is reached. This creates a condition of $N_2 = 2\ N_1$.

Figure 4 illustrates this condition, in which the output noise power contributed by the excess noise source exactly equals the sum of the amplified input termination noise plus the receiver noise contribution. Since this excess noise ratio was adjusted with the attenuator to be equal to input termination noise plus receiver noise (thereby causing $N_2 = 2\ N_1$), from equation (5) it can be seen that the attenuated excess noise ratio is equal to the noise figure of the receiver. In the case of an argon source, it can be read as 15.2 db minus the attenuator setting (in db).

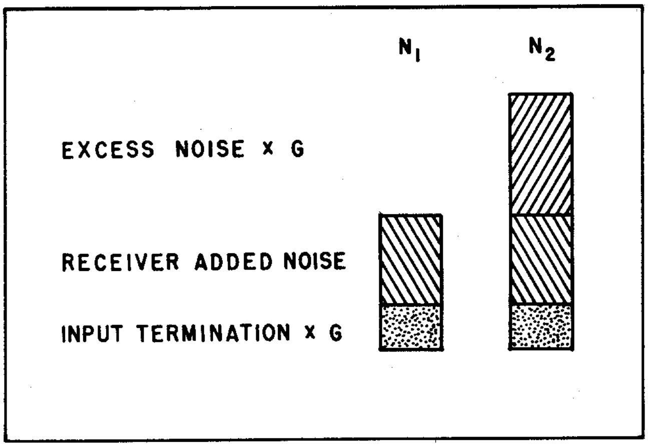

Figure 4. Representation of total noise power output for the "twice-power" method of manual noise figure measurement.

While the attenuator reduces the amount of excess noise injected into the system, it has no effect on input termination noise power if the termination and attenuator are at the same temperature, since, regardless of the amount of attenuation, when the excess noise source is cold the receiver input is still looking at a matched input at temperature T.

B. "Y-Factor" Method of Noise Figure Measurement

A method closely resembling the "Twice-Power" method involves the determination of the numerical ratio N_2/N_1 (which is called Y-Factor) and the calculation of noise figure by substitution in equation (5).

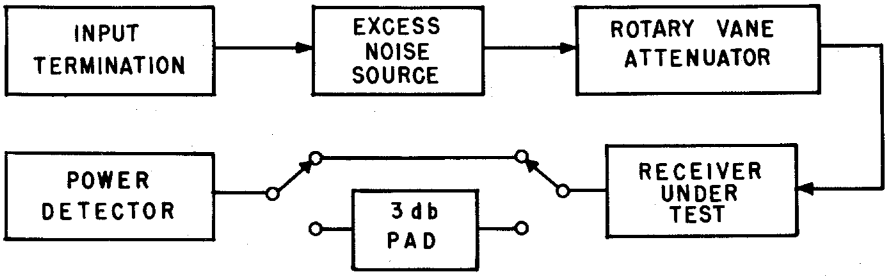

Figure 3. The "twice-power" method of manual noise figure measurement.

In practice, the Y-Factor method generally makes use of an IF attenuator with a power indicator set to a convenient reference. The IF attenuator change in going from source OFF to source ON then yields the Y-Factor, which is then entered in the equation. Graphs are also available for specific values of relative excess noise, with co-ordinates calibrated in "Y-Factor" and noise figure. (See figure 5.)

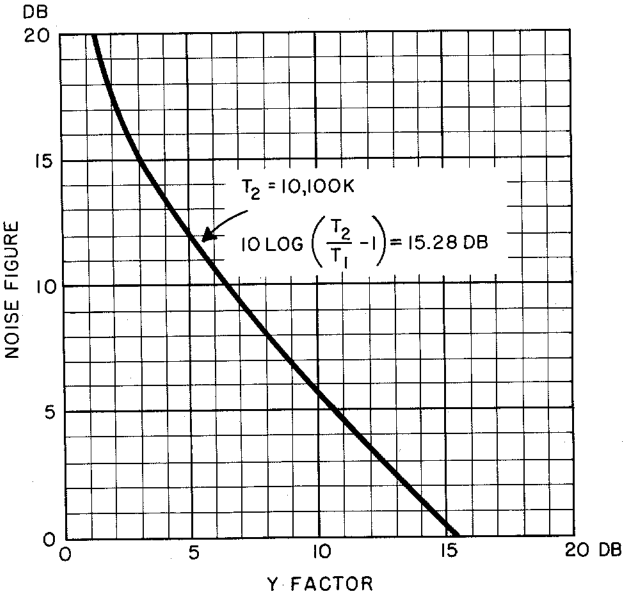

Figure 5. A "Y-Factor" chart for determining noise figure with a 15.28 db excess noise source.

C. Automatic Noise Figure Measurements

While manual measurements yield valid results, they still represent a tedious process...and are often not easily accomplished by unskilled personnel. And since there is not a continuous indication of noise figure, the work of reducing noise figure is considerably slowed by the necessity of a new measurement after each circuit change.

To fill the need for a direct-reading, continuously indicating noise figure meter, at least three automatic systems have been devised. All depend on the periodic insertion of known excess noise into the system. This results in a pulse train of 2 pulse levels, N_2 and N_1. The pulse train typically is amplified in an IF strip and then separated into two distinct levels by selective gating. These levels, together with the amount of excess noise insertion, contain the information needed to directly indicate the noise figure on a meter face.

The three automatic systems differ in their method of indication. One approach uses a special ratio resolving meter movement which responds to the ratio of the two signal levels in a manner similar to a wattmeter movement. Such a meter movement is quite expensive, however, and in general has not achieved wide acceptance in this country.

Another method uses AGC in its IF amplifier to hold constant the value $(N_2 + N_1)/2$ and measures $N_2 - N_1$. This contains the necessary information to measure noise figure, and is especially useful in very high noise figure cases, where N_1 is close in value to N_2. In such cases, the difference of the levels is more definitive than their ratio.

A third approach actually measures the ratio N_2/N_1 and displays this ratio on a meter face calibrated by the equation:

$$F_{db} = 15.2 - 10 \log (N_2/N_1 - 1)$$

Such an instrument is shown in simplified block diagram form in figure 6.

In operation, the gating source pulses the noise source at a rate of 500 cps; N_1 and N_2 pulses arrive at the IF amplifier. Noise sources have a finite noise build-up time, so the IF amplifier is gated to pass only the final amplitudes of N_1 and N_2 to the square law detector. The detected N_2 pulse is switched to an AGC integrator, where a voltage for gain control of the IF amplifier is derived. The time constant of this circuit is made long enough to control the IF amplifier gain even when the N_1 pulse is passing through it. Since the AGC action keeps the detected N_2 pulse at a constant level, a measurement of the detected N_1 pulse is, in effect, a measurement of the pulse ratio. The N_1 pulse is measured by switching it to the meter integrator and meter.

Convenient internal calibration of the meter is accomplished by artificially creating readings of "$+\infty$" and "$-\infty$". By pulsing the noise source during both the N_2 and N_1 time periods, we obtain a condition of $N_2 = N_1$. In the formula $F_{db} = 15.2 - 10 \log (N_2/N_1 - 1)$ this condition results in a noise figure of $+\infty$. The artificial condition of $F = -\infty$ would correspond to an "N_1" value of "0". This can be created by gating "off" the IF amplifier during the "N_1" time period. If the metering circuit is designed to be a linear indicator of the power of "N_1" (square law detector) and the meter minimum position is calibrated as $-\infty$ and the full scale deflection as $+\infty$, all other points on the meter face can be calculated by the formula $F_{db} = 15.2 - 10 \log (N_2/N_1 - 1)$. For example, an "$N_1/N_2$" ratio of 1/2 would bring about a mid-scale reading. From the formula this mid-scale reading is calculated to be 15.2. In a similar fashion the balance of the scale is calibrated.

Figure 7 is an actual meter face from an automatic noise figure meter. In addition to calibration for 15.2 db excess noise sources, it is calibrated for 5.2 db temperature-limited diode sources. The linear current scale is used in adjusting noise source exitation current.

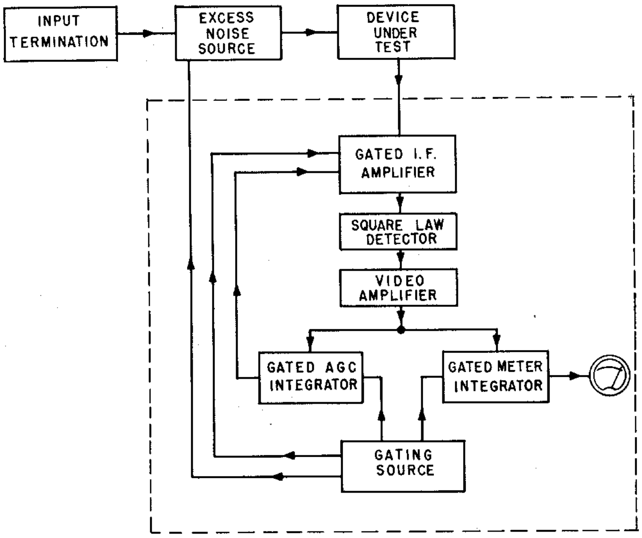

Figure 6. Simplified block diagram of automatic noise figure measurement system.

NETWORKS IN CASCADE

The effects of the noise contribution of networks in cascade can be seen in figure 8. This illustration shows the input termination and three networks, each with gain G and noise figure F. The power graphs assume that each network is active with gain greater than 1. However, the analysis will be equally valid for passive networks with gain less than 1.

The input termination supplies a noise power kTB which is amplified by the three networks and appears at the output as kTB $G_1G_2G_3$.

The noise contribution of network 1 (by equation 4) is $(F_1 - 1)$ $kTBG_1$. When further amplified by networks 2 and 3, the noise power appears at the output as $(F_1 - 1)\ kTG_1G_2G_3$. Similarly, the output noise contributed by network 2 is $(F_2 - 1)\ kTBG_2G_3$; and from network 3, $(F_3 - 1)\ kTBG_3$. The system noise figure (F_s) is the ratio of actual noise power output to noise power output contributed by the input termination.

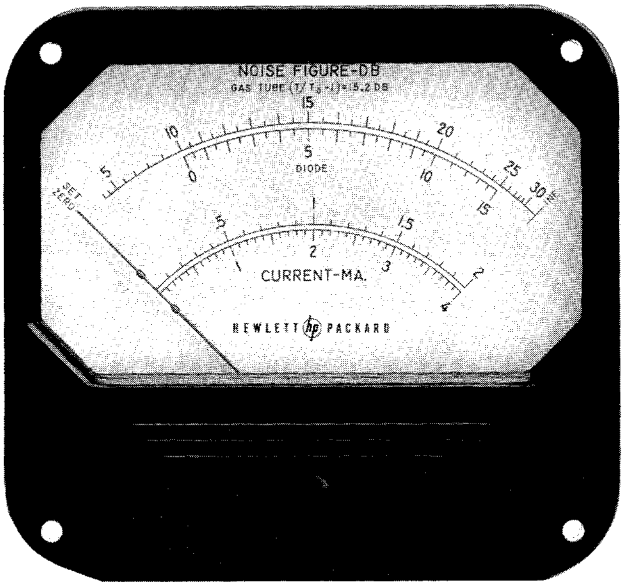

Figure 7. Meter face from an automatic noise figure meter.

$$F_s = \frac{\text{Total noise output}}{kTBG_1G_2G_3}$$

$$= \frac{kTBG_1G_2G_3 + (F_1 - 1)kTBG_1G_2G_3}{kTBG_1G_2G_3}$$

$$+ \frac{(F_2 - 1)kTBG_2G_3 + (F_3 - 1)kTBG_3}{kTBG_1G_2G_3}$$

$$= F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1G_2}$$

The general equation for the noise figure of networks in cascade is then

$$F_s = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1G_2} + \ldots + \frac{(F_n - 1)}{G_1G_3 \ldots G_{(n-1)}}$$

It can be seen that overall noise figure of any cascaded amplifying system depends primarily on noise figure of the first stage. The effects of subsequent stages is reduced by the gain up to that point. The use of a passive stage with gain less than 1, on the other hand increases the importance of the subsequent stage's noise figure.

ACCURACY CONSIDERATIONS

Up to this point we have deliberately ignored several possible sources of error. Actual measurement techniques must consider the possibility of system errors caused by mismatch, temperature, and image and spurious responses. The instrument accuracy of the noise source and noise figure meter should also be considered.

A. Temperature

In the derivation of equation (5),

$$F_{db} = 10 \log \frac{(T_2 - T_o)}{T_o} - 10 \log \left(\frac{N_2}{N_1} - 1 \right)$$

the ambient temperature was assumed to be 290, hence T_o cancelled out of all terms except the figure "10 log $(T_2 - T_o)/(T_o)$". T_2 is the equivalent fired temperature of the noise source. In specifying the relative excess noise power of a noise source, the manufacturer knows the value of T_2 and rates the tube in terms of the standard temperature, 290^oK (62.6^oF). A variation of 20^o from the assumed 290^o, for example, would cause an error of about 0.3 db in measured noise figure.

B. Mismatch

Noise power obeys all power transfer laws. But since it is random in phase, mismatches cause ambiguous errors rather than known amounts of power loss. In the automatic noise figure meter measuring the ratio of N_2/N_1, a mismatch affecting both pulses does not affect accuracy, since the ratio remains unchanged.

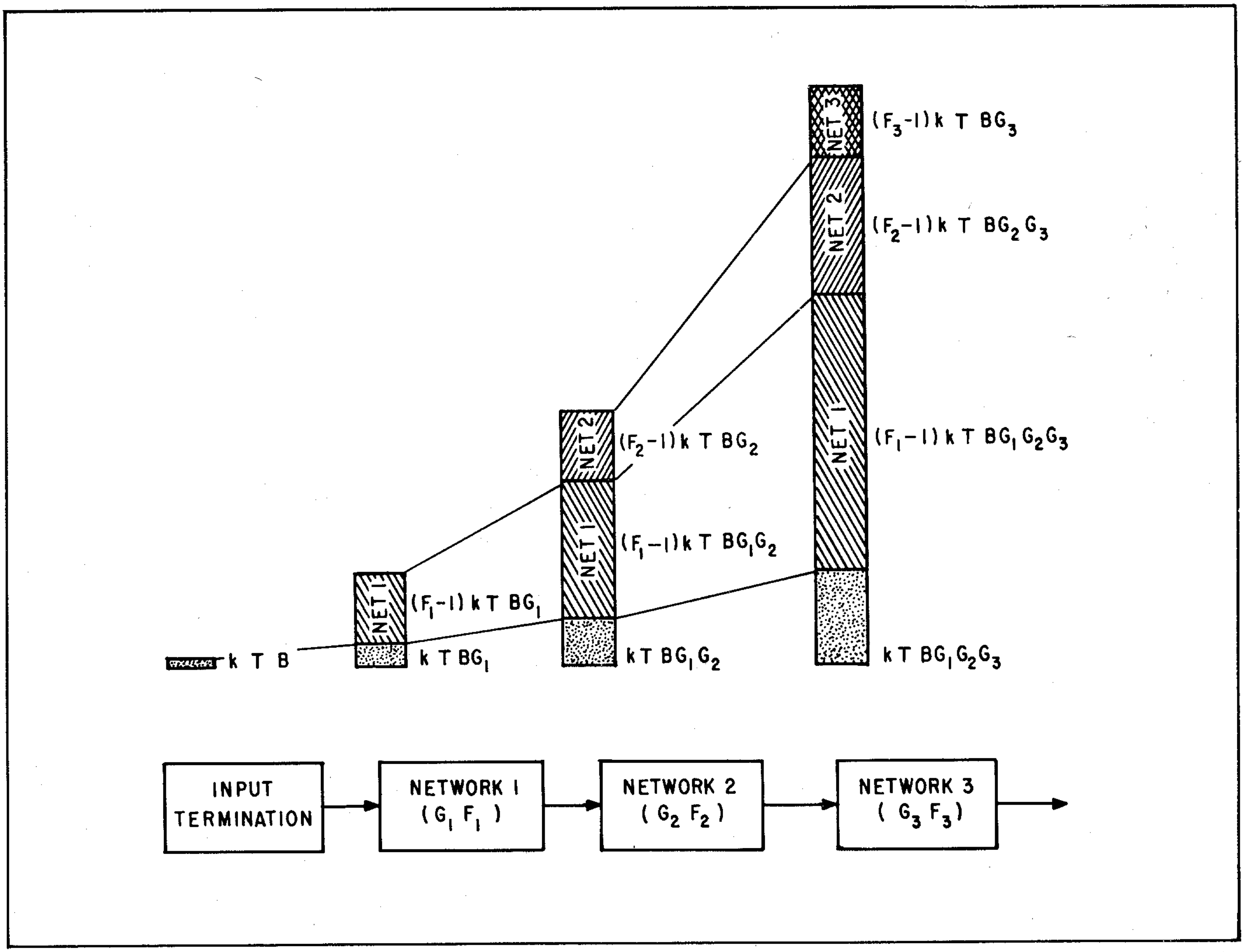

Figure 8. The effects of noise contribution of networks in cascade.

The critical matching situation, then, involves the excess noise source. Noise sources are rated in available excess noise power; thus mismatches will cause an ambiguous amount of excess noise power to be coupled to the system. Figure 9 shows the effect of several possible conditions of mismatch.

Note that the possible error is maximum at low noise figures where the greatest accuracy is usually desired. The importance of well-matched noise sources over the entire frequency range of interest is apparent. Hewlett-Packard waveguide sources are rated as 1.2 maximum, and typically are better than 1.1. hp Diode sources are 1.3 or better over most of their rated band.

C. Image and Spurious Response

In using a broadband excess noise source, an automatic noise figure meter measures the true noise figure of the total pass band of the device under test. If, in its operation, the device does not utilize the full pass band for signal information (as would be the case of a radar receiver with an image response) its operating noise figure will be higher than the measured noise figure. This apparent noise figure can be calculated by the equation:

$$\text{Operating } F_{db} = F_{db} \text{ (reading)} + 10 \log (B_t/B_u),$$

where B_t is the total bandpass of the device, and B_u is the operational bandpass.

This equation is a convenient simplification that assumes constant gain in the device under test.

D. Excess Noise Source Accuracy

At the present time, accuracy of pulse type gaseous discharge noise source tubes is specified ± 0.5 db, for the National Bureau of Standards has not yet provided a certification for pulse-operated tubes. However, NBS does offer certification for cw operation at 9.0, 9.8, and 11.2 GHz to ± 0.1 db. The Red Bank Division of Bendix Corporation, Eatontown, N.J., also offers cw certification, with some additional uncertainty over that of NBS.

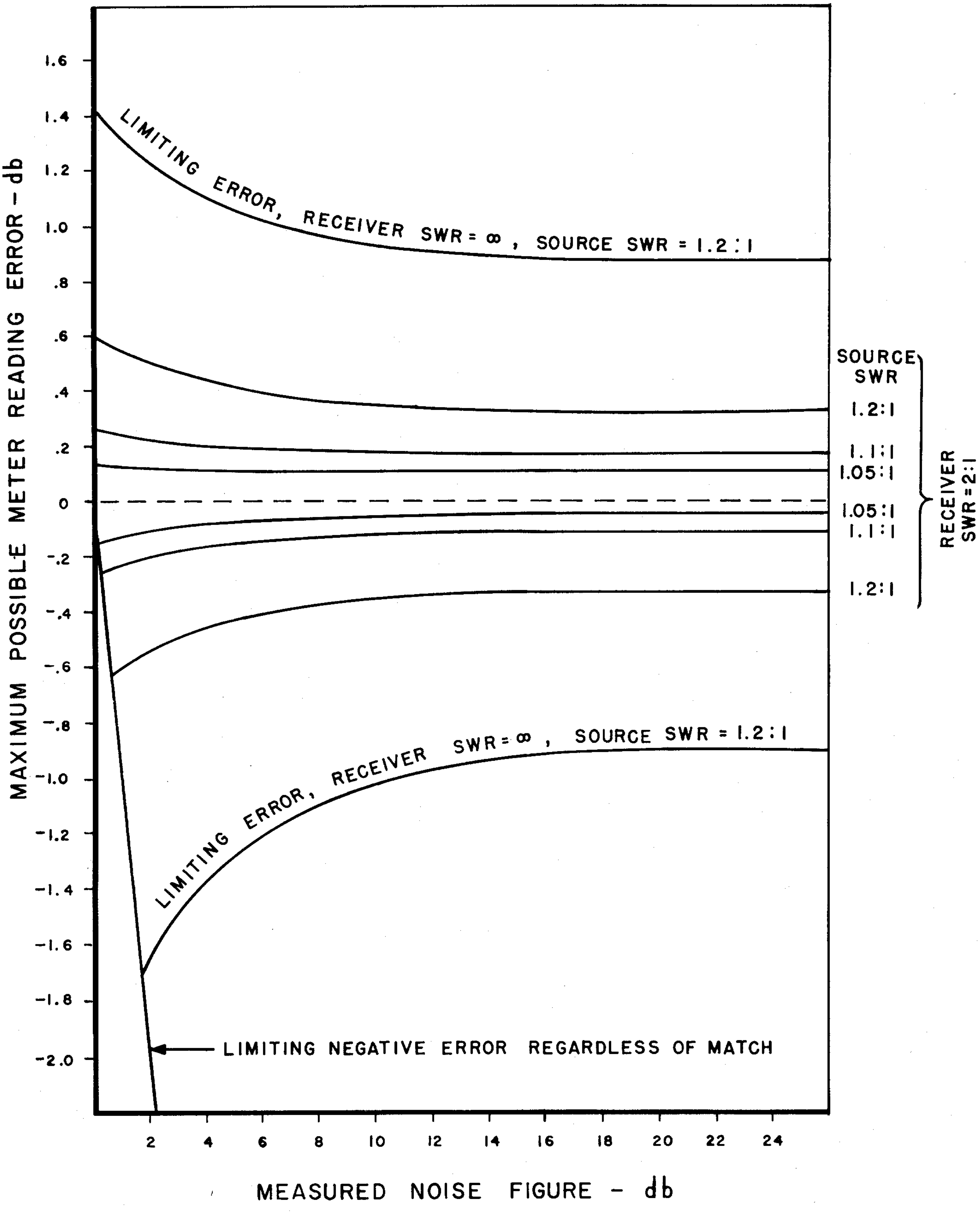

Figure 9. Typical error effects for several possible conditions of mismatch between noise source and receiver.

Temperature-limited diode sources can be made to much closer specifications, but are limited to the vhf region and below.

Correction of the excess noise ratio of a noise source is sometimes necessary to take into account conditions of temperature varying from the standard 290°K (see page 6). This is especially true of diode sources where the heat of the temperature-limited diode tends to raise the ambient temperature of the output resistor.

E. Noise Figure Meter Accuracy

The automatic noise figure meters manufactured by the Hewlett-Packard Company are specified accurate within ±1/2 db over most of their range, ±1 db over the remainder. This accuracy specification includes the effects of meter tracking, variation from square law, and aging effects. With the current specifications of noise sources, and the possible errors caused by mismatch and temperature, such meter accuracy would seem to be consistent with overall system accuracy.

For measuring low-noise devices, a modified noise figure meter which provides increased resolution and accuracy is available on special order. The modification expands a 4-db portion of the scale (up to 6 db) over the full range of the meter scale. Noise figure meter accuracy in the expanded mode of operation is ± 0.2 db; either expanded or normal operation can be selected.

NOISE FIGURE INSTRUMENTATION

Hewlett-Packard has the following equipment presently available for making automatic noise figure measurements.

Model 340B Noise Figure Meter. When used with an hp noise source, automatically measures and continuously displays the noise figure of IF or rf amplifiers tuned to 30 or 60 MHz and of radar or microwave receivers with intermediate frequencies of 30 and 60 MHz. (Collectively, hp noise sources cover frequencies from 10 MHz to 18 GHz.)

Model 342A Noise Figure Meter. Is similar to hp Model 340B except that it operates on five frequencies between 30 and 200 MHz. Four of these frequencies are 60, 70, 105 and 200 MHz, the fifth is the basic 342A tuned amplifier frequency of 30 MHz.

Model 343A VHF Noise Source. Specifically for IF and rf amplifier noise measurement, a temperature-limited diode source with broadband noise output from 10 to 600 MHz.

Model 345B IF Noise Source. Operates at either 30 or 60 MHz, as selected by a switch (other frequencies between 10 and 60 MHz on special order). Another selector permits matching 50, 100, 200 and 400 ohm impedances.

Model 347A Waveguide Noise Sources. Argon gas discharge tubes mounted in waveguide sections. For all frequencies from 3.95 through 18.0 GHz, providing uniform noise throughout range; maximum swr of 1.2.

Model 349A UHF Noise Source. Argon discharge tube for automatic noise figure readings on scatter communications receivers, L-band radars, parametric amplifiers, or other devices 400 to 4,000 MHz. The source can also be furnished with a neon discharge tube.

For detailed information and specifications on any of these noise figure devices, please contact your nearby Hewlett-Packard field engineer.