

PhotoStress® Instruments

Tech Note TN-706-1

Corrections to Photoelastic Coating Fringe-Order Measurements

1.0 Introduction

All instrumentation systems have in common two intrinsic characteristics which inevitably limit their accuracy in varying ways and degrees: (1) the tendency to respond to other variables in the environment in addition to the variable under investigation, and (2) the tendency to alter the variable being measured. The user of the reflection polariscope should keep in mind the fact that the photoelastic coating method is not an exception to the foregoing generalities.

There are two generally applicable approaches to minimizing the ever-present instrumentation errors. The first of these involves the careful design, planning, and execution of the measurement by a knowledgeable test engineer so as to compensate for, and/or control, those error sources which are subject to such treatment. Following this, the test engineer must employ the second defense against errors, which is to apply corrections to the measurement as appropriate. Both of these approaches can and should be utilized in the process of strain measurement with photoelastic coatings.

The principal sources of error in the photoelastic coating method are as follows:

- 1. Parasitic (initial) birefringence
- 2. Reinforcement effects in plane-stress systems
- 3. Reinforcement and strain-extrapolation effects for plates in bending
- 4. Temperature effects

All of the above are manifestations of the inherent error propensities of measurement systems in general; and all of these can be treated by the approaches already given for minimizing errors. Procedures for treating each of the error sources are provided in the following sections.

2.0 Parasitic Birefringence

Any initial color pattern in a photoelastic coating (prior to applying test loads) causes an error in subsequent fringe-order measurements which must be corrected. Under normal circumstances, residual birefringence in the coating is produced only by severe mishandling of the plastic during or after application to the test object. In such cases, it is usually preferable to strip off the coating and apply a new one, rather than attempting to make corrections for the parasitic birefringence.

There are instances, however, when the nature of the test circumstances may unavoidably introduce residual birefringence, and necessitate correction of the fringe-order measurements. Following are several examples:

A. Residual birefringence caused by a difference between the temperature at which the coating was bonded in place and the test temperature.

This parasitic birefringence is produced by differential thermal expansion between the coating and the test object (see Section 5.0). The initial birefringence is concentrated at free edges and decreases with distance from the edge. For homogeneous, isotropic test materials, it approaches zero at distances greater than four times the coating thickness. At points far removed from the edges, the stress state in the plane of the coating due to differential thermal expansion is equal biaxial stress $(\sigma_1 = \sigma_2)$, and produces no birefringence.

B. Parasitic birefringence due to contraction of the cement.

Over periods of a month or so, the cement used to bond the coating to the test part may continue to polymerize and, in so doing, contract. The effect is similar to that in Item A, and is concentrated at the edges.

C. Edges not protected against humidity.

If the edges of the plastic coating are not protected from humidity by a layer of cement, some moisture may be absorbed through the finished edges of the plastic. The result will be swelling of the plastic along the edges, producing parasitic birefringence in these areas.

In all of the foregoing cases, the residual birefringence is localized near the edges; and, if the edge of the coating matches the edge of the test object, a very simple correction procedure can be applied. At every point on the free (unloaded) boundary of a test object, the principal axes are tangent and perpendicular to the boundary (the boundary itself is an *isostatic*, or principal stress, trajectory). This is



equally true of the stress at the edge of the coating, whether caused by test loads or by effects A, B, and C. Because the load-induced and parasitic birefringences are congruent, direct superposition can be employed, and correction can be made at all points on the free boundary by simply subtracting the measured fringe order under no load from that measured with the test loads applied. It is important to note that direct linear superposition of stress states is permissible only when the directions of the principal stresses coincide for the two states of stress.

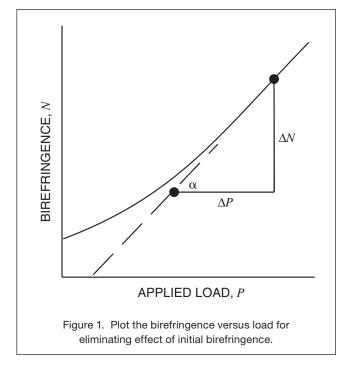
When residual birefringence exists in the coating due to mishandling of the plastic, or to yielding of the test part after it has been coated, the directions of the principal stresses causing the parasitic birefringence will not generally coincide with the principal axes produced by the test loading. In such cases, the correction cannot be made by simple subtraction, and other methods must be employed.

Before proceeding, it is necessary to test whether the directions of the principal axes associated with the parasitic birefringence coincide with those under loading. For this purpose, first trace or closely observe the isoclinic patterns of the parasitic birefringence (with no load applied to the test part). Then load the part, and examine the isoclinics again. If the isoclinic patterns under load and no-load conditions are identical (i.e., the isoclinics do not move as load is applied), both states of stress have the same principal axes, and the parasitic birefringence can be subtracted algebraically from that "caused by" or "introduced with" load. If the isoclinics move with the application of load, one of the two following procedures can be performed.

Figure 1 is representative of the relationship between applied load and observed birefringence when the principal axes of the parasitic birefringence differ in orientation from those due to load. As shown by the figure, the effect of the initial birefringence decreases as the load increases. Therefore, at a load sufficiently high to cause the graph to approach linearity, the slope of the curve gives the relationship between the increment in load, ΔP , and the corresponding increment in load-induced birefringence, ΔN . As a first approximation, the birefringence due to load can be calculated for any load from the equation represented by the tangent line:

$$N = P \tan \alpha = P \frac{\Delta N}{\Delta P}$$
 (1)

Obviously, if the magnitude of the parasitic birefringence is large, or the misalignment of principal axes is great, the plot will not approach linearity closely, and this approximation cannot be used. In such cases, or whenever



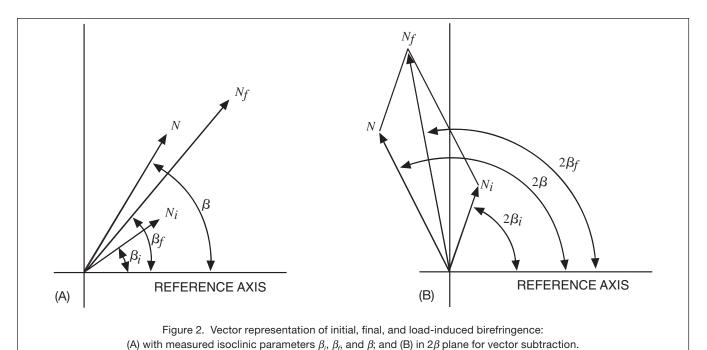
greater accuracy is required, the correction can be made analytically by the vector-subtraction method described in the following.

Referring to Figure 2A, assume that an initial parasitic birefringence, N_i , has been measured and represented as a vector at the angle β_i from the horizontal reference axis. The length of the vector is proportional to the fringe order, N_i , and the direction corresponds to the direction of the major (algebraically greater) principal stress causing the initial birefringence. After applying the specified test loads, a final birefringence, N_f , is observed, for which the isoclinic parameter of the major principal stress is β_f . The magnitude and direction of N_f reflect the combined effects of both the initial and the load-induced birefringences.

When the vectors for N_i and N_f are replotted with the angles doubled (at $2\beta_i$ and $2\beta_f$, respectively) as in Figure 2B, it can be shown that the vector for the birefringence, N, due only to the test load, can be obtained by subtracting N_i from N_f vectorially in the 2β plane. From the geometry of the diagram in Figure 2B, the magnitude of the load-induced birefringence becomes:

$$N = \sqrt{N_f^2 + N_i^2 - 2N_f N_i \cos 2(\beta_f - \beta_i)}$$
 (2)





 $(N_c \sin 2\beta_c - N_c \sin 2\beta_c)$

major principal stress is:

and the angle between the horizontal reference axis and the

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{N_f \sin 2\beta_f - N_i \sin 2\beta_i}{N_f \cos 2\beta_f - N_i \cos 2\beta_i} \right)$$
(3)

The procedure used to correct for parasitic birefringence is as follows:

- 1. With no load on the test object, measure the fringe order, N_i , of the initial parasitic birefringence at the test point, and the isoclinic parameter, β_i , of the *major principal stress*. (For instructions on determining which principal axis has the algebraically maximum principal stress, see the reflection polariscope instruction manual.)
- 2. After application of the test load, again measure the fringe order (N_f) and the isoclinic angle (β_f) to the axis of the major principal stress. Note that these measurements result from the combination of parasitic and load-induced birefringences.
- 3. Calculate the corrected fringe order, *N*, resulting only from the test load, from Equation (2).
- 4. Calculate the isoclinic angle, β , between the reference direction and the axis of the load-induced major principal stress from Equation (3).

Example

Assume that the following measurements resulted from Steps 1 and 2 of the foregoing procedure (these are the data plotted in Figure 2):

$$N_i = 1.37$$
 ; $\beta_i = 35^{\circ}$
 $N_f = 3.42$; $\beta_f = 50^{\circ}$

From Equation (2), the corrected fringe order due only to load is:

$$N = \sqrt{(3.42)^2 + (1.37)^2 - 2 \times 3.42 \times 1.37 \times \cos 2(50 - 35)}$$

N = 2.33 (note that direct algebraic subtraction would have produced the answer N = 2.05)

And the angle between the reference axis and the axis of the major principal stress is, from Equation (3):

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{3.42 \sin(100) - 1.37 \sin(70)}{3.42 \cos(100) - 1.37 \cos(70)} \right)$$
$$\beta = \frac{1}{2} \tan^{-1} (-2.043)$$
$$\beta = -32^{\circ}, \text{ or } +58^{\circ}$$



One of these angles corresponds to the algebraically maximum principal stress and the other to the minimum. Referring to Figure 2B, and recalling that N is the vector difference between N_f and N_i , it is evident that 2β is somewhat greater than 90°, and therefore 58° is the proper choice for β .

3.0 Reinforcement Effects in Plane-Stress Problems

The term "plane stress" is generally applied to structural members such as plates and panels which are loaded only in their midplanes, and are not subject to significant out-of-plane bending moments. Thin-walled pressure vessels and certain other structures can also be treated *approximately* as plane-stress problems.

When a plane-stress structural member to which a photoelastic coating has been bonded is subjected to loads, the coating reinforces the member and carries part of the load. As a result, the strains in the test member are lower than they would be without the coating present. The reinforcement error is very small for metal structures, and can often be ignored. However, when the test object is made from plastics or other nonmetals, the error is generally significant, and correction is required.

As shown in Reference 7, the correction relationship for reinforcement effects in plane-stress situations can be expressed as:

$$C_{PS} = 1 + E^* t^*(4)$$

where:

 C_{PS} = factor by which the observed fringe order in plane stress must be multiplied to obtain the corrected fringe order

 $E^* = \frac{E_c}{E_s}$ = ratio of the elastic modulus of the photoelastic coating to that of the test specimen

 $t^* = \frac{t_c}{t_s}$ = ratio of the coating thickness to the specimen thickness

Equation (4) is plotted in Figures 3A and 3B (broken lines) for five different materials. Figures 3A shows C_{PS} for epoxy photoelastic coatings where E_c = 420,000 psi [2.9 GPa] as in Micro-Measurements Types PS-10, PL-1 and PL-10. Figure 3B shows CPS for polycarbonate photoelastic coatings where Ec = 360,000 psi [2.5 GPa] as in Micro-Measurements Type PS 1. It can be seen from the figures that C_{PS} is always greater than unity because the reinforcing effect of the photoelastic coating causes the observed fringe order to be too small. When correcting photoelastic measurements, C_{PS} can be read directly from the graph for the materials represented there, or it can be calculated from Equation (4) for any material of known elastic modulus.

When loading is applied by in-plane **fixed displacement**, the strain field is the same for coated and uncoated test parts. Consequently, there is no plane-stress correction.

4.0 Reinforcement and Strain Extrapolation Effects in Bending

A. Applied Bending Moments

When thin beams, plates, or shells are subjected to bending moments, the effects of the photoelastic coating on the structural member are generally much greater than for the plane-stress case; and a *correction is almost always required*. The influence of the coating on a member in bending is quite complex, and the correction factor must account for three different effects as follows:

- 1. The neutral axis of the coated member is shifted toward the coated side.
- 2. The coating increases the stiffness of the member, and decreases the deformation (curvature) for a particular applied bending moment.
- 3. There is a strain (and fringe-order) gradient through the thickness of the coating. The polariscope measures the average fringe order at the midplane of the coating, which is further from the neutral axis than the surface of the test member.

The first two of the above effects tend to depress the observed fringe order compared to the correct value, and the third tends to exaggerate the fringe-order indication. All three effects operate simultaneously, but are influenced differently by the elastic-modulus and thickness ratios, E^* and t^* . A single correction factor for all three effects is presented by Zandman, et al. in Reference 7, and is given here in a re-expressed form:

$$C_B = \frac{1 + E * (4t * + 6t *^2 + 4t *^3) + E *^2 t *^4}{1 + t *}$$
 (5)

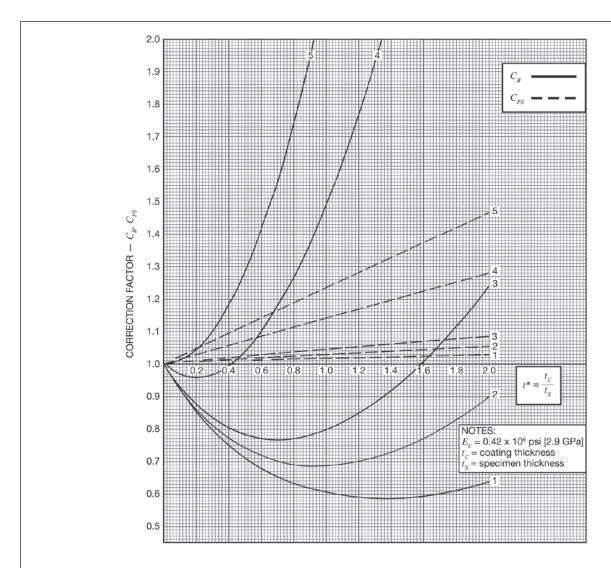
where:

 C_B = factor by which the observed fringe order in bending must be multiplied to obtain the corrected fringe order

 E^* , t^* = elastic-modulus and thickness ratios as defined for Equation (4)

Equation (5) is plotted in Figures 3A and 3B (solid lines) for the same specimen materials and photoelastic coating used in evaluating Equation (4). Because the strain-exaggeration effect is predominant with high-elastic-modulus materials, it can be seen from the figure that for such materials the





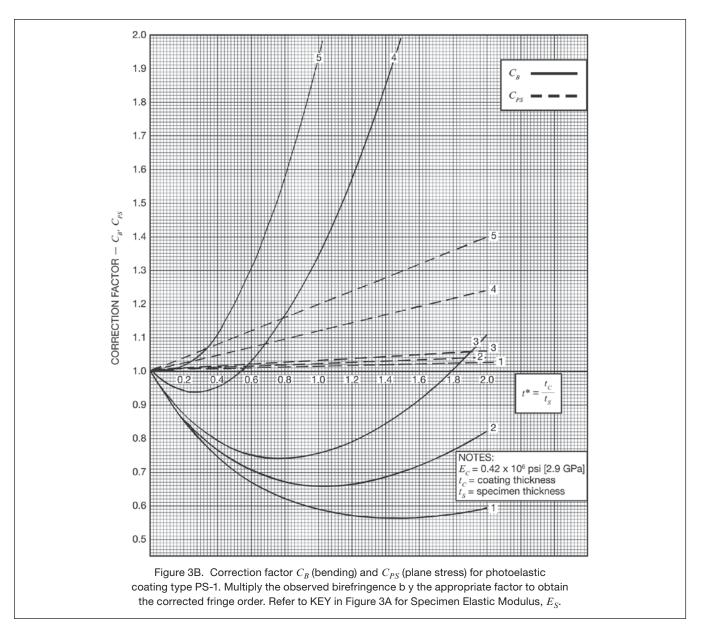
Key	Specimen Elastic Modulus, E_s	
	in 10 ⁶ psi	in GPa
1. Steel	30.0	207.0
2. Cast Iron	16.0	110.0
3. Aluminum	10.0	69.0
4. Reinforced Plastic	3.0	21.0
5. Wood	1.8	12.5

Figure 3a. Correction factor C_B (bending) and C_{PS} (plane stress) for photoelastic coating Types PS-10, PL-1 and PL-10. Multiply the observed birefringence by the appropriate factor to obtain the corrected fringe order.

observed fringe order is usually too high, and must be multiplied by a factor less than unity. With low-elastic-modulus materials, the stiffening effect of the coating predominates, and the measured fringe order is commonly too low, requiring a correction factor greater than unity.

In addition to their uses in correcting fringe-order measurements, Figures 3A and 3B should also be referred to as a guide for selecting the thickness of photoelastic coatings. When feasible, it is preferable to select the coating thickness so that C_B falls into one of four areas:





Area A: The coating thickness is small compared to the specimen thickness, resulting in correction factors close to unity.

Area B: The correction factor C_B passes through a minimum, indicating that the ratio of the observed to the corrected fringe order is a maximum. This thickness selection is advantageous when expected strain magnitudes are very low, and all available sensitivity is needed.

Area C: Correction factor C_B is unity. This selection, which for structural metals leads to a coating thickness greater than the plate thickness, is particularly appropriate

for very thin plates. Also, in cases of combined bending and plane stress, with the ratio of the two components unknown, it limits the error to that caused by the planestress reinforcement.

Area D: Correction factors C_{PS} and C_B are identically equal. When the ratio of bending to plane stress is unknown, the single common correction factor can be applied directly to the measured fringe order.

NOTE: Equations (4) and (5) should be used to calculate C_{PS} and C_B when the elastic moduli of the coatings differ from the values used in Figures 3A and 3B.



B. Imposed Bending or Flexural Deformation

The correction factor (C_{BA}) for imposed flexural deformation differs from the correction factor (C_B) for applied bending moment loading. Bending to a predetermined deformation requires discrete radii of curvatures in the deformed part. This is illustrated in Figure 4 where the imposed radii of curvature in the uncoated and coated parts are shown to be equal, $R_o = R_c$. Indeed, the flexural rigidity of the coated part is greater than the uncoated part and, consequently, a larger bending moment is required to deform the coated part, $M_c > M_o$. Simple geometric consideration of Figure 4 provides:

$$C_{B\Delta} = \frac{1 + E^* t^*}{1 + t^*}$$
 (6)

Equation (6) is plotted in Figure 5 for the same five specimen materials considered in Figures 3A and 3B. The broken lines in Figure 5 show C_{BA} for the polycarbonate photoelastic coating (Type PS-1 where $E_c \approx 0.36 \times 106$ psi [2.5 GPa]). The solid lines reflect C_{BA} for the epoxy family of photoelastic coatings (Types PS-10, PL-1, and PL-10 where $E_c \approx 0.42 \times 10^6$ psi [2.9 GPa]). It is clear, from Figure 5 that C_{BA} is relatively independent of the type of photoelastic coating applied to metal structures. However, C_{BA} is dependent on the photoelastic coating type when the test objects are made from lower modulus plastics. Equation (6) should be used when the elastic modulus of the coating differs from the values used in Figure 5.

C. Correction Examples – Bending and Plane Stress

Case 1: Thin Member – Applied Bending Moment

An aluminum alloy cantilever beam of cross-section dimensions $\frac{1}{8} \times 1$ in [3 x 25 mm] is coated with Type PS-1

photoelastic plastic having a thickness equal to the beam thickness – that is,

$$t* = 1.0$$

From Figure 3B, the bending correction factor:

$$C_B \approx 0.75$$

The measured (uncorrected) fringe order at a specified test point on the beam is $\hat{N} = 1.40$; and the corrected fringe order is thus:

$$N = C_B \hat{N} = 0.75 \times 1.40 = 1.05$$

Assuming that the photoelastic plastic has a fringe value, f, of 725 μ in/in [μ m/m] per fringe,

$$\varepsilon_x - \varepsilon_v = f \times N = 725 \times 1.05 = 761 \,\mu\text{in/in} [\mu\text{m/m}]$$

Case 2: Thin Member — Imposed Bending Deformation

The same cantilever aluminum alloy beam of *Case 1* is subjected to a predetermined end displacement rather than bending moment loading. As for *Case 1*, $t^* = 1.0$ and from Figure 5:

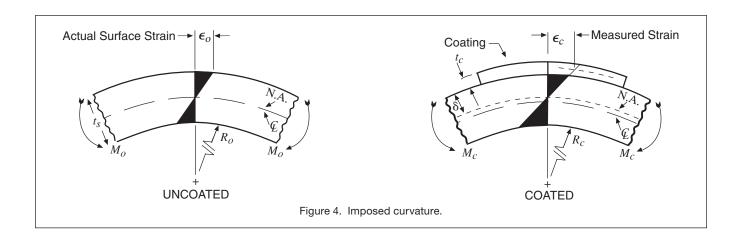
$$C_{B\Delta} \approx 0.52$$

The measured (uncorrected) fringe order at a specified test point on the beam is N = 1.9; and the corrected fringe order is thus:

$$N = C_{B\Lambda} \hat{N} = 0.52 \times 1.9 = 0.99$$

As for Case 1, the fringe value (f) is 725 μ in/in [μ m/m] per fringe,

$$\varepsilon_{\rm x} - \varepsilon_{\rm v} = f \times N = 725 \times 0.99 = 718 \,\mu{\rm in/in} \,[\mu{\rm m/m}]$$





Case 3: Biaxial Stress Field

A large-diameter cylindrical steel tank is subjected to internal pressure. The state of stress ($\sigma_1 = 2\sigma_2$) corresponds very nearly to plane stress, with negligible bending present. In this instance, only the plane-stress correction factor (broken lines in Figure 3A) need be applied. Considering that the tank wall thickness is 0.625 in [15.8 mm] and the Type PL-10 coating thickness is 0.125 in [3.2 mm], what is the plane-stress correction factor, C_{PS} ?

For a coating-to-specimen thickness ratio, $t^* = t_c/t_s = 0.2$, Figure 3A shows that $C_{PS} \approx 1.0$, and no correction is necessary.

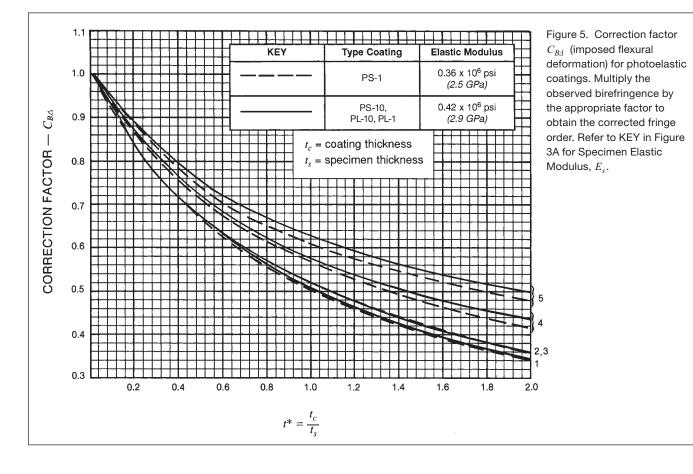
Case 4: Indeterminate Combination of Plane Stress and Bending

A study is undertaken to determine the state of surface stress in a thin aluminum-alloy diaphragm subjected to an unknown combination of membrane and bending stresses. The thickness of the diaphragm is 0.060 in [1.5 mm]. What thickness of epoxy photoelastic coating

should be selected to render $C_B = C_{PS}$? This unique correction factor requirement is satisfied, on Figure 3A, at the point where the two curves for aluminum (No. 3) intersect. This occurs where $t^* \approx 1.72$ and $C_B = C_{PS} \approx 1.07$ at the point of intersection. The coating thickness required is:

$$t^* = 1.72$$
 $t_c = 1.72 \times 0.06 \approx 0.103 \text{ in } [2.62 \text{ mm}]$

As another example, consider a steel diaphragm which is 0.020 in [0.5 mm] thick, and find the thickness of epoxy photoelastic coating so that $C_B = 1.0$. It is evident from Figure 3A that the coating-to-specimen thickness ratio t^* for this case (solid curve No. 1) must be greater than 2.0, and is probably between 3.0 and 3.5. Solving Equation (5) successively with different values of t^* and plotting the results gives a value of $t^* = 3.4$ ($t_c = 0.068$ in [1.73 mm]) for $C_B = 1.0$. From Equation (4) the plane-stress correction factor C_{PS} is 1.05. Applying a correction factor of 1.025 to the measured birefringence will limit the error to $\pm 2.5\%$ for any ratio of membrane to bending stress.





5.0 Correction for the Effects of Temperature Changes

Whenever the temperature changes during a test, a system of stresses will develop in the plastic coating as a result of the difference in thermal expansion coefficients between the structure and the plastic. Under certain circumstances, this condition can produce errors in the observed fringe orders, and necessitate corrective measures. Although a full treatment of thermal effects is given in Reference 6, the following procedures will be adequate for most practical applications.

A. Regions Not Located on Boundaries (at distances from the edges of the coating greater than four times the coating thickness)

At interior points, away from the edges, the effect of differential expansion between the coating and the structure is to produce a plane state of stress ($\sigma_1 = \sigma_2$) in the coating. Whether or not a correction must be introduced depends upon whether a normal- or oblique-incidence measurement is made.

Normal incidence: In normal incidence, a *plane* state of stress, for which the principal stresses in the coating are identically equal, causes no birefringence. Therefore, no correction is necessary, and the observed fringe orders are used directly to determine the mechanical and/or thermal stresses in the structure, assuming that it is fabricated from a homogeneous, isotropic material.

Oblique incidence: When the coating is observed in oblique incidence, the temperature-induced stress state produces a difference in the *secondary* principal stresses (in the plane perpendicular to the oblique light ray). The result is a superimposed birefringence (or "zero-shift") which is calculable from:

$$N_0(\Delta T) = \frac{1}{f} \left(\frac{1 + v_c}{1 - v_c} \right) \frac{\sin^2 \theta}{\cos \theta} (\alpha_s - \alpha_c) \Delta T \tag{7}$$

where:

 $N_0 (\Delta T)$ = birefringence due to temperature change only (irrespective of the state of stress in the structure)

 v_c = Poisson's ratio of photoelastic coating

 α_s , α_c = thermal expansion coefficients of structure and coating, respectively

 θ = angle of oblique incidence (from the surface normal)

 ΔT = change in temperature

Since $N_0(\Delta T)$ is independent of direction in the plane of the coating, correction can be made by algebraic subtraction of the zero-shift from the observed fringe order. That is,

$$N_{\theta} = \hat{N}_{\theta} - N_0 \left(\Delta T \right) \quad (8)$$

where:

 N_{θ} = corrected birefringence (in oblique incidence)

 \hat{N}_{θ} = observed (uncorrected) birefringence, including temperature-induced zeroshift

 $N_0(\Delta T)$ = birefringence due to temperature change only (irrespective of the state of stress in the structure)

B. Free Edges and Boundaries

Because there is only one nonzero principal stress on any free edge or boundary, oblique-incidence measurements are not made there. In normal incidence, however, fringes will appear at the edges due to temperature change; and, for a particular temperature change, the sign and magnitude of the temperature-induced birefringence depend upon the sign and magnitude of the edge curvature.

The most effective procedure for this case is to employ a dummy specimen having the same configuration as the test part, coated with the same plastic in the same thickness, and subjected to the same thermal environment, but always left free of mechanical and thermal stresses. The measured birefringence on the dummy specimen at any temperature then represents the temperature-shifted zero for measurements on the actual test part. Because the direction of the only nonzero principal stress at a free boundary is always tangent to the boundary, irrespective of what caused the stress, the correction is made by direct subtraction, in the manner of Equation (8).

The same dummy specimen can be used in the same manner to obtain the zero-shift for oblique-incidence measurements at interior points, away from the edges. Because the stress state induced at interior points by a temperature change is plane ($\sigma_1 = \sigma_2$) and has no directional properties, direct correction can again be made with Equation (8).

In those cases where the actual coated test part can be subjected to the test temperature while remaining free of thermal and mechanical stresses, no separate dummy is necessary. Zero readings can be made at the test temperature — either on the boundary in normal incidence, or at interior points in oblique incidence — and the correction performed with Equation (8) or its equivalent.



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Technical References

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Tech Note TN-704, "How to Select Photoelastic Coatings".

Tech Note TN-708, "Principal Stress Separation in Photo-Stress® Measurements".

Application Note IB-221, "Instructions for Casting and Contouring PhotoStress® Sheets".

Application Note IB-223, "Instructions for Bonding Flat and Contoured PhotoStress® Sheets to Test-Part Surfaces".