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on supplementary comparison

VNIIM/KRISS Bilateral Comparison of DC Magnetic Flux Density
by Means of a Transfer Standard Coil

FINAL REPORT

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1. Introduction

The purpose of these bilateral comparisons is the conformance of the base unit of magnetic measurements - a DC magnetic flux density and its ratio to a current, reproduced in VNIIM and KRISS - at institutes where adequate conditions for precise measurements in low magnetic fields are provided and the appropriate equipment for to attain a high level of accuracy is available.

Special conditions are required to ensure the best uncertainty levels of the measurements, and these were fulfilled for this comparison. These conditions are resumed in the presence of the following factors: Non-magnetic laboratory located far from sources of human-produced magnetic interference, automatic system of deep compensation of the Earth's magnetic field, and quantum measuring instrument for precision conversion of a highly uniform magnetic flux density is reproduced by transfer Standard Coil to AC frequency. As a result of this bilateral comparison, total standard uncertainties of tesla/ampere(T/A) measurements of $(1-2) \times 10^{-6}$ were expected.

This comparison may form a basis for follow-on multilateral APMP comparison of DC magnetic flux density standards.

2. Participants and schedule

2.1 Participants

D. I. Mendeleev Institute for Metrology(VNIIM)
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Korea Research Institute of Standards and Science(KRISS)
Daejeon, Republic of Korea

2.2 Quantity

DC Magnetic Flux Density by Means of a Transfer Standard Coil

2.3 Comparison schedule

Table 1. Comparison schedule.

Year	Date of Measurement	Laboratory	Country or Economy
2010	22 June – 24 June	KRISS(Solenoid No.3)	Korea (Rep. of)
	23 June – 24 June	KRISS(Transfer std. coil, VNIIM No.1)	Korea (Rep. of)
	6 July – 7 July	VNIIM(Transfer std. coil, VNIIM No.1)	Russia

3. The traveling standard

The transportable T/A and Tesla standard ЭСТВ №01, was developed and fabricated by VNIIM on the basis of the quartz Garret solenoid [1] designed to produce a known magnetic flux density (MFD) and a known ratio of magnetic flux density to DC current. A single layer two-pitch solenoid of the Garret type with a winding installed in a screw groove cut on the surface of a cylindrical quartz carcass has a length of about 400 mm and a diameter of 130 mm. The travelling standard is capable of carrying a maximum current in solenoid winding I_C of 1.05 A, which produces a field in its centre with a MFD B_0 of about 1 mT. The field in the solenoid is homogeneous within 2×10^{-6} in the central spherical region of 30 mm diameter, which is sufficient to enable a narrow atomic magnetic resonance (AMR) line width for precision MFD measurements up to 1 mT [2].

A copper thermo resistor of the value $R_t \approx 125 \Omega$ attached to the internal surface of a solenoid quartz carcass for use as a coil temperature reference. By varying the room temperature, the temperature coefficient of the coil conversion coefficient (coil constant) $K_B^{R_{t0}} = B_0/I_s$ with respect to the change of R_t was measured at VNIIM using an He-Cs AMR standard magnetometer, and comparisons with the primary standard solenoid VNIIM which value of a constant is determined on the dimension of a solenoid winding.

Results of long-term VNIIM researches of stability of a travelling standard (solenoid ЭСТВ № 01) from a current and the temperature dependence of a coil constant K_B are represented as (1):

$$K_B^{R_t} = 9.539846 [1 + 8.2 \times 10^{-7} (125.0 \Omega - R_t)] \times 10^{-4} \text{ T/A} \quad (1)$$

Estimation of total standard uncertainty ($k=1$) is $u_c = 5 \times 10^{-7}$.

Table 2. Coil constant from a current and the temperature dependence.

Date	I (A)	R_t (Ω)	K_B ($\times 10^{-4}$ T/A)
22.05.09	0.6	123.5	9.5398578
21.08.09	0.6	123.8	9.5398558
13.05.10	0.6	124.7	9.5398466
14.05.10	0.6	124.8	9.5398494
21.05.09	1.0	125.4	9.5398425
20.08.09	1.0	126.7	9.5398341
17.05.10	0.6	126.7	9.5398312
14.05.10	1.0	128.0	9.5398221

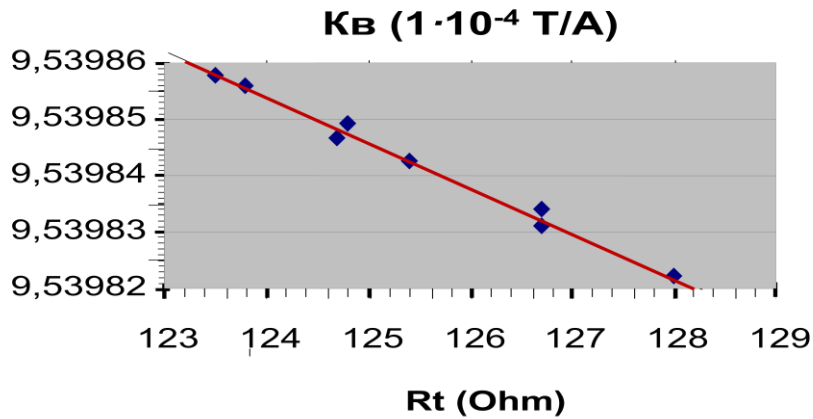


Figure 1. Measurement of solenoid temperature coefficient.

4. Measurement method

4.1 Measurements and reproduction of the magnetic flux density

Measurements and reproduction of a MFD field of a solenoids were carried out at KRISS and VNIIM in space with full compensation of a Earth magnetic field (EMF) and its variations. The measuring systems for the EMF compensation consist of a large volume, three-component coil system (TCS) with active control of coil currents by precision DC current sources and external optically pumped atomic magnetic resonance converter.

Compensation of the EMF variations is controlled in the TCS system by means of a converter placed inside an auxiliary conjugated coil system (“two-volume system”).

Transfer standard coil constant T/A in its geometrical center is derived from the results of direct measurements by standard He-Cs magnetometer of the MFD B_S generated by a solenoid at the applied DC current I in the solenoid winding.

Measurements were carried out at two opposite directions of the current in the winding of the solenoid to remove the effect of the uncompensated part of the Earth magnetic field. Solenoid constant K_B is determined as(2):

$$K_B = B_S / I, \quad K_B = \frac{B_{K1} + B_{K2}}{I} \quad (2)$$

where B_{K1} and B_{K2} are the values of MFD measured by a standard magnetometer at two opposite directions of current.

All results of MFD measurements are based on the newest gyromagnetic ratio of the shielded protons γ'_P value recommended by CODATA in 2010[3], and the experimental determination of the ratio ($\gamma_{4\text{He}} \gamma'_P$) of ^4He atoms to protons [4] as(3):

$$B_{\text{KR}} = (2\pi/\gamma_{4\text{He}}) \cdot f_{4\text{He}} \quad \text{and} \quad B_{\text{KR}} = K_{\text{VN}}^{\text{B}} \cdot I_{\text{KR}} \quad (3)$$

where $f_{4\text{He}}$, B_{KR} and I_{KR} are ^4He AMR frequency, MFD and current, respectively, measured by KRISS, K_{VN}^{B} – conversion constant of VNIIM solenoid measured by VNIIM.

4.2 Comparison of the conversion constants of VNIIM and KRISS solenoids

The conversion constants of each solenoid (VNIIM and KRISS) were measured with identical current in their windings. The ratio of the coil constants of the KRISS solenoids (K_{KR}^{B}) and VNIIM (K_{VN}^{B}) is defined as(4):

$$[K_{\text{B}}^{\text{KR}} / K_{\text{B}}^{\text{VN}}] = [B_{\text{KR}} / B_{\text{VN}}] \quad (4)$$

where B_{KR} and B_{VN} are measured values of MFD in VNIIM and KRISS solenoids.

Type *B* uncertainty(systematic) of current measurements in this case is excluded. One series of measurements is carried out in *N* pairs of single measurements of MFD K_{B}^{KR} and K_{B}^{VN} , B_{K} and current *I*. Each pair of measurements is performed at two opposite directions of the current *I*. On the basis of the results of these measurements the average value of results of measurements and their uncertainty were calculated.

5. Comparison results

Results of measurements in KRISS of VNIIM solenoid conversion coefficient and magnetic flux density.

5.1 Input data for applied standards of electric and magnetic units and constants.

- Transported standard quartz solenoid VNIIM– ЭСТВ №1

$$K_{\text{B}}^{\text{Rt}} = 9.539846 [1 + 8.2 \times 10^{-7} (125.0 \Omega - R_t)] \times 10^{-4} \text{ T/A}, \quad u_c = 5 \times 10^{-7} (k=1)$$

- Standard quartz solenoid KRISS - №3

$$K_{\text{B}} = 12.310614 [1 - 1.65 \times 10^{-6} (50.0 \Omega - R_t)] \times 10^{-4} \text{ T/A}, \quad u_c = 2 \times 10^{-7}$$

- Standard He-Cs magnetometer KRISS #1 (SM)

$$B_{SM} = (2\pi/\gamma_{4He}) f_{4He}; 2\pi/\gamma_{4He} = 28.023802 \text{ Hz/nT}, u_c = 4 \times 10^{-8}$$

$$\gamma'_P = 2.675153268 \times 10^{-8} \text{ s}^{-1}\text{T}^{-1}, u_c = 2.5 \times 10^{-8} [3]$$

$$(\gamma_{4He}/\gamma'_P) = 658.200555, u_c = 3 \times 10^{-8} [4]$$

- Voltage standard, Fluke, type 7004, #41037

$$E_0 = 1.018\,052\,7 \text{ V}, u_{E0} = 2 \times 10^{-7}$$

- Resistance 10 Ω , Tettex, type 3275, #3012749, $t_{oil} = 25.04^\circ\text{C}$

$$R_{10} = 10.000005 \Omega, u_{R0} = 3.5 \times 10^{-7}$$

- Multimeter, Fluke, type 8508A, # 3012749, $u_{M0} = 4 \times 10^{-7}$

- Current reproduced in windings of solenoids

$$I_{KR} = [(E_0 - 0.0180000)/R_{10}] = [(1.0180527 - 0.0180000)/10.000005] = 0.10000522 \text{ A}$$

$$u_{I(KR)} = (u_{E0}^2 + u_{R0}^2 + u_{M0}^2)^{1/2} = 6 \times 10^{-7}$$

5.2 Measurement results

Table 3. The results of measurement of MFD and coil constant.

Date	Solenoid, ЭСТБ №1, VNIIM						$u_A (k=1)$
	$B_S(\text{nT})$	$u_A(B_S), (v_i=5)$ 1×10^{-6}	K_{Bt} (10^{-4} , T/A)	$R_{ii}(\Omega)$	$\Delta K_{Bt} =$ $K_{B0} - K_{Bi}$ (nT/A)	$K_{B0} (10^{-4}\text{T/A})$ $R_t = 125.00 \Omega$	
23.06. 2010	95403.58	0.8	9.539862	125.70	0.05	9.539867	
23.06. 2010	95403.48	0.3	9.539852	125.72	0.05	9.539857	
24.06. 2010	95403.48	0.5	9.539852	125.84	0.06	9.539857	
24.06. 2010	95403.52	0.5	9.539856	125.60	0.07	9.539859	
Average	95403.52	0.2	9.539856	125.72 /125.00	0.06	9.539860	$u_A = 2 \times 10^{-7}$

Total standard uncertainty ($k=1$) of the measured value of the B_{VN_KR} magnetic flux density unit is $u_c = 4.7 \times 10^{-7}$. MFD reproduced VNIIM solenoid using KRISS current and measured by KRISS standard He-Cs AMR magnetometer is given(5):

$$B_{KRISS} = (95403.52 \pm 0.05) \text{ nT} \quad (5)$$

Uncertainty budgets of measurements by KRISS of the VNIIM magnetic flux density using KRISS current (Table 4).

Table 4. Uncertainty budgets.

Items of uncertainty	Type of estimations	Distribution	Standard uncertainty	Degree of freedom
Standard He-Cs magnetometer KRISS	B	Normal	3×10^{-7}	∞
Variations of an external field and current	A	Normal	2×10^{-7}	3
MFD non-uniformity	B	Normal	3×10^{-7}	∞
Total standard uncertainty		$u_c = 4.7 \times 10^{-7}$		

Magnetic flux density calculated through value of a solenoid VNIIM conversion coefficients K_{VN} and value of a current I_{KR} in its winding, reproduced in KRISS, by the values of used volt and resistance standards, is equal to(6):

$$B_{VN_KR} = K_{VN} \times I_{KR} = (953984.6 - 0.06) \text{ nT/A} \times 0.10000522 \text{ A} = (95403.40 \pm 0.09) \text{ nT}, \quad (6)$$

where 0.06 nT/A is correction to K_{VN} for solenoid temperature.

Uncertainty of the KRISS current: $u_{I(KR)} = (u_{E0}^2 + u_{R0}^2 + u_{M0}^2)^{1/2} = 6 \times 10^{-7}$ and VNIIM solenoid $u_{VN_SOL} = 5 \times 10^{-7}$, $u_{VN_KR} = 7.8 \times 10^{-7}$. Total standard uncertainty of the reproduced value of the B_{VN_KR} magnetic flux density is 9.3×10^{-7} or 0.09 nT.

The relative difference ∂B reproduced at VNIIM and measured at KRISS MFD values is received (7):

$$\partial B = [(B_{VNIIM_KRISS} - B_{KRISS}) / B_{KRISS}] = 0.8 \times 10^{-6} \text{ (0.08 nT)} \quad (7)$$

Experimental data of VNIIM/KRISS comparison of the MFD unit have shown an agreement with a total standard uncertainty of 1.2×10^{-6} .

5.3 Results of KRISS solenoid conversion coefficient measurements

Table 5. Conversion coefficient of solenoid.

Date	Solenoid №3, KRISS						$u_A(k=1)$
	B_S n(T)	$u_A(B_S)$ ($v_i=5$),	K_{Bb} 10^{-4} T/A	R_{ti} , Ω	$\Delta K_{Bi} = K_{B0}$ K_{Bi} , nT/A	K_{B0} , 10^{-4} T/A	

		1×10^{-6}	R_{ti}, Ω			$R_t=50.00 \Omega$	
22.06. 2010	123112.72	0.6	12.310631	50.34	0.07	12.310638	
22.06. 2010	123112.60	0.7	12.310619	50.25	0.05	12.310624	
23.06. 2010	123112.58	0.4	12.310617	50.23	0.05	12.310622	
24.06. 2010	123112.70	0.6	12.310629	50.28	0.06	12.310635	
Average	123112.65	0.2	12.310624	50.00	0.06	12.310630	$u_A=2 \times 10^{-7}$

Total standard uncertainty ($k=1$) of the measurements at value of the KRISS and VNIIM solenoids constants is 7.6×10^{-7} (Table 6). Measured in KRISS ratio VNIIM/KRISS solenoids conversion coefficients with use the KRISS standard helium-caesium magnetometer and 100 mA current reproducing system is the following(8):

$$(K_{KR}/K_{VN})_{comp10} = (123106.08/95398.41) = 1.29044096 \tag{8}$$

Budget of uncertainty of measurements at the values of the KRISS and VNIIM solenoids constants (Table 6).

Table 6. Uncertainty budgets of solenoids constants.

Items of uncertainty	Type of estimations	Distribution	Standard uncertainty	Degree of freedom
Standard He-Cs magnetometer	B	Normal	3×10^{-7}	∞
Current VNIIM	B	Normal	6×10^{-7}	∞
Variations of an external field and current	A	Normal	2×10^{-7}	3
MFD Non-uniformity	B	Normal	3×10^{-7}	∞
Total standard uncertainty		$u_c = 7.6 \times 10^{-7}$		

Total standard uncertainty ($k=1$) of the VNIIM/KRISS coils constant ratio is $u_c = 4.7 \times 10^{-7}$ (uncertainty in the reproduction of the current is excluded). The value of the KRISS solenoid constant calculated on the basis of the measured ratio of two solenoids constants and the constant of VNIIM solenoid is(9):

$$K_{KR(comp10)} = K_{VN} \times (K_{KR}/K_{VN})_{comp10} = 1231060.2 \text{ nT/A } (R_t = 50.00 \Omega) \tag{9}$$

with total standard uncertainties of 9.4×10^{-7} .

Presented KRISS preliminary value of the solenoid constant is(10):

$$K_B = 123106.14 [1 - 1.65 \times 10^{-6}(50.0 - R_t)] \text{ nT/A} \quad (u_c = 2 \times 10^{-6}) \quad (10)$$

to be in agreement with the above result of comparisons (difference is 1.1×10^{-6}) but has 2 times bigger uncertainty of measurements.

The relative difference in MFD values ∂B reproduced by VNIIM in KRISS and measured at KRISS is equal to 1.2×10^{-6} . It shows the agreement of the results of measurements to estimation a component of their uncertainty.

5.4 Measurements conversion coefficient of VNIIM solenoid at VNIIM after measurements at KRISS

Input data for applied standards of electric and magnetic units and constants.

- Transported standard quartz solenoid VNIIM– ЭСТВ №1

$$K_B^{\text{Rt}} = 9.539846 [1 + 8.2 \times 10^{-7} (125.0 \Omega - R_t)] \times 10^{-4} \text{ T/A}, \quad u_c = 5 \times 10^{-7}$$

- Standard helium-cesium magnetometer VNIIM ЭТ-1 (SM), $u_c = 3 \cdot 10^{-7}$

$$B_{\text{SM}} = (2\pi/\gamma_{4\text{He}}) f_{4\text{He}}; \quad 2\pi/\gamma_{4\text{He}} = 28.023802 \text{ Hz/nT} \quad (u_c = 3 \times 10^{-8})$$

$$\gamma'_{\text{P}} = 2.675153268 \times 10^{-8} \text{ s}^{-1}\text{T}^{-1} (2.5 \times 10^{-8})[3], \quad (\gamma_{4\text{He}}/\gamma'_{\text{P}}) = 658.200555(3 \times 10^{-8})[4]$$

- Voltage standard – Fluke, type 732B, # 7590806, $E_0 = 1.01812628 \text{ V}$, $u_{E_0} = 1 \times 10^{-7}$

- Resistance 1 Ω , type M3024, # 1, $t_{\text{oil}} = 25.04^\circ\text{C}$

$$R_0(1 \text{ A}) = 0.99998804 \Omega, \quad u_{R_0} = 0.5 \times 10^{-7}$$

- Multimeter, Agilent, type 3458A, # MY 45044017, $u_{M_0} = 2.5 \times 10^{-7}$

- Current reproduced in windings of solenoids

$$I_{\text{VN}} = [(E_0 - 0.0180000)/R_0] = [(1.01812628 - 0.0180000)/0.99998804] = 1.0001382 \text{ A}$$

$$u_{I(\text{VN})} = (u_{E_0}^2 + u_{R_0}^2 + u_{M_0}^2)^{1/2} = 2.7 \times 10^{-7}$$

Results of measurements of the VNIIM solenoid conversion coefficient (Table 7) and magnetic flux density using VNIIM standard magnetometer. Budget of uncertainty in measurements of the VNIIM

solenoid magnetic flux density (Table 8). Budget of uncertainty of measurements at value of the VNIIM solenoid constant (Table 9).

Table 7. Measurement of VNIIM solenoid conversion coefficient.

Date	Solenoid, ЭСТВ №1, VNIIM						$u_A(k=1)$
	B_S (nT)	$u_A(B_S)$, ($\nu_i=5$) 1×10^{-6}	K_{Bt} (10^{-4} , T/A)	B_S (nT)	$u_A(B_S)$, ($\nu_i=5$) 1×10^{-6}	K_{B0} (10^{-4} T/A $Rt=125.00 \Omega$)	B_S (nT)
06.07.2010	954115.30	0.2	06.07.2010	954115.30	0.2	06.07.2010	954115.30
06.07.2010	954115.18	0.04	06.07.2010	954115.18	0.04	06.07.2010	954115.18
06.07.2010	954115.37	0.1	06.07.2010	954115.37	0.1	06.07.2010	954115.37
06.07.2010	954115.48	0.07	06.07.2010	954115.48	0.07	06.07.2010	954115.48
07.07.2010	954116.02	0.1	07.07.2010	954116.02	0.1	07.07.2010	954116.02
Average	954115.47		Average	954115.47		Average	954115.47

Table 8. Uncertainty budgets of VNIIM solenoid magnetic flux density.

Items of uncertainty	Type of estimations	Distribution	Standard uncertainty	Degree of freedom
Standard He-Cs magnetometer	B	Normal	3×10^{-7}	∞
Variations of an external field and current	A	Normal	0.5×10^{-7}	4
MFD non-uniformity	B	Normal	3×10^{-7}	∞
Total standard uncertainty		$u_c = 4.2 \times 10^{-7}$		

Table 9. Uncertainty budgets of VNIIM solenoid constant.

Items of uncertainty	Type of estimations	Distribution	Standard uncertainty	Degree of freedom
Standard He-Cs magnetometer	B	Normal	3×10^{-7}	∞
Current VNIIM	B	Normal	2.7×10^{-7}	∞
Variations of an external field and current	A	Normal	2×10^{-7}	3
MFD	B	Normal	3×10^{-7}	∞

nonuniformity				
Total standard uncertainty		$u_c = 5.4 \times 10^{-7}$		

Averaged values of solenoid magnetic flux density and conversion coefficient measured by standard magnetometer are 954115.47 nT and 9.5398463×10^{-4} T/A, respectively. The total standard uncertainties are 4.2×10^{-7} and 5.4×10^{-7} .

The calculated value of the VNIIM magnetic flux density using VNIIM solenoid and current is(11):

$$B_{VN} = K_{VN} \cdot I_{VN} = [953984.60 - 1.07] \text{ nT/A} \times 1.0001382 \text{ A} = 954115.37 \text{ nT} \quad (11)$$

The total standard uncertainty of B_{VN} value is 5.7×10^{-6} . The relative difference ∂B reproduced by VNIIM solenoid and current and measured by standard magnetometer values was determined to be(12):

$$\partial B = [(B_{KB, I} - B_{4He})/B] = 1.0 \times 10^{-7} (0.10 \text{ nT}) \quad (12)$$

The experimental data of VNIIM/KRISS comparison of the MFD unit have shown an agreement with a total standard uncertainty of 1.0×10^{-6} .

6. Conclusion

The above reported results have shown a good agreement of experimental comparison data with an estimation of the uncertainty components of applied standards. The coordinated values of the MFD unit and its ratios to DC current at the level of standard uncertainty $(1-1.2) \times 10^{-6}$ ($k=1$) on the basis of gyromagnetic ratio of the shielded protons γ'_P value, recommended by CODATA in 2006, the experimental determination of the ratio (γ_{4He}/γ'_P) of ^4He atoms to protons and standards of a current of two institutes are received.

In contrast to the key comparison of CCEM.M.-K1(2000) the results in this report not only comparisons of unit T/A, but also T, reproduced by two institutes contain. The difference between the units T/A reproduced by VNIIM and KRISS in this comparisons, is lowered from 8×10^{-6} to 1×10^{-6} , while VNIIM result in KC of 2000, was the closest (8×10^{-7}) coincided with average weighed value T/A according to these multilateral comparisons. The received results give a basis for carrying out multilateral comparisons of standard quantum magnetometers of metrological institutes in the framework of APMP with participation of geomagnetic observatories, which require an establishment of the unified standard of the MFD unit.

The results of this comparison show the possibility of decreasing the uncertainty of the MFD unit determination at direct comparisons of standard quantum magnetometers, that assumes an exception of uncertainty of reproduction and measurement of the solenoids conversion constant and current.

7. References

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