

## **Final Report**

**Bilateral Supplementary Comparison: P1-APMP.EM.RF-S4**

Calibration factor of thermistor mount power sensors:  
in Type-N, 30 MHz – 3000 MHz

Participants:  
Measurement Standards Laboratory of New Zealand  
National Measurement Laboratory (Australia)

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## Summary

This is the final report on the APMP bilateral supplementary comparison in RF power measurement P1-APMP.EM.RF-S4. The purpose of the comparison is to determine the level of consistency of calibration results given by two national standards laboratories.

This is a comparison of one of the high-frequency key quantities. The comparison protocol was based on that used in the key comparison CCEM.RF-K8, however, the frequency points differ. One of the participants, the National Measurement Laboratory, also took part in CCEM.RF-K8.

The travelling standard is a Hewlett-Packard 8478B thermistor mount, with a type-N male RF connector. The calibration factor is determined at a number of frequencies between 30 MHz and 3000 MHz, together with an appropriate statement of uncertainty. Measurements have been made at a nominal power level of 1 mW. The value of the reflection coefficient is also determined, as it is needed for the uncertainty calculation.

The pilot laboratory was the National Measurement Laboratory, in Australia, and the comparison coordinator was the Measurement Standards Laboratory of New Zealand.

This report contains a brief description of the measurement setups at each laboratory and a summary of the associated uncertainty budgets. The actual measurements from each laboratory are presented as they appear in calibration certificates from the respective laboratories.

# 1 Participants and organisation

## 1.1 Participants

The contact person at the pilot laboratory for this comparison was:

Dr. Tieren Zhang  
National Measurement Laboratory CSIRO  
Division of Telecommunications and Industrial Physics  
P.O. Box 218 LINDFIELD  
NSW 2070 AUSTRALIA

Tel.: + 61 2 9413 7273  
Fax: + 61 2 9413 7202  
e-mail: [tieren.zhang@csiro.au](mailto:tieren.zhang@csiro.au)

The contact person at the coordinating laboratory was:

Dr. Blair Hall  
Measurement Standards Laboratory of New Zealand  
Industrial Research Ltd.  
P.O. Box 31-310  
Lower Hutt 6009  
New Zealand

Tel.: +64 4 931 3198  
Fax: +64 4 931 3194  
e-mail: [b.hall@irl.cri.nz](mailto:b.hall@irl.cri.nz)

## 1.2 Schedule

The following schedule was proposed for this comparison.

1. The travelling standard will first be measured by NML, then by MSL.
2. MSL will ensure that the standard is delivered to NML on or before 7 July 2003.
3. It is expected that four weeks will suffice for NML to complete measurements and return the standard to MSL. MSL will then measure the standard and draft the report proposal.
4. It is expected that the report proposal will be sent to NML before 8 September 2003.

The comparison commenced on time and measurements were performed at NML according to schedule. The travelling standard was received at MSL in early August, however delays then occurred and the measurements at MSL were performed in December, 2003. The first draft of the report was sent to NML in January, 2004. A full report was submitted to the Chair of the APMP-TCEM on March 19, 2004. Revised versions were later submitted on April 27 and June 22.

## 2 Measurement Setups

### 2.1 MSL Measurement Setup

A resistive power splitter was used to deliver power from an RF generator to two measurement arms. The nominal power delivered to each arm was 1 mW. A calibrated thermistor mount and the unknown mount were connected alternately to one arm, while the other arm was connected to a reference thermistor. Readings at the reference arm were used to normalise readings at the other arm, correcting for any fluctuations in source output level.

The MSL standard thermistor mount has been calibrated at NML.

The thermistor mounts were operated with a pair of Larsen (NBS type-IV) self balancing DC-substitution RF bridges, built at MSL [1]. The bridges were connected to a dual-channel digital voltmeter.

A 8478B thermistor mount is fitted with two identical dual-element thermistor sensors, one of which can be used for temperature compensation. However, the Larsen bridge does not use the temperature compensating element, and relies instead on carrying out RF power measurements in a temperature stable environment. The measurements were carried out in a temperature controlled environment at  $20 \pm 1^\circ\text{C}$ . The thermistor heads were placed in insulating material to protect them from drafts and slow down any temperature changes. Humidity was not controlled.

Measurement of the voltage reflection coefficients for both the calibrated mount and the unknown mount were performed using an Agilent 8753ES vector network analyser. The measurement of effective source match for the resistive power splitter was made using same network analyser and the ‘direct method’ of Juroshek [2].

### 2.2 NML Measurement Setup

A type N power splitter system was used in this comparison. A reference thermistor mount was connected to one of the output ports of the splitter and the device under test (DUT) was connected to the other output port. Nominal power at the output ports was 1 mW. The calibration factor of the DUT is derived using the method of [3]. This method explicitly accounts for the full measured complex scattering matrix of the splitter and the measured complex reflection coefficients of the reference and

DUT sensors. In particular the ratio  $S_{31}/S_{21}$  of the splitter has been measured using a number of repeated connections.

The thermistor mounts were operated with a pair of NML Dual Precision self balancing bridges. Each bridge was connected to a digital voltmeter. The temperature compensating elements were used.

The reference thermistor mount was calibrated using the NML Micro-calorimeter.

Measurements of the reflection coefficients for both the reference mount and the DUT were performed using HP 8510C and Advantest R3762A vector network analysers.

### 3 Measurement Results

#### 3.1 MSL Measurement Result

The measured values of calibration factor and reflection coefficient of the travelling thermistor mount are shown in Table 1.

TABLE 1

Frequency (MHz)	Calibration factor (Note 1)		Reflection coefficient (linear)			
			Magnitude $ \Gamma $		Phase (Degree)	
Value	Value	Uncertainty (Note 2)	Value	Uncertainty (Note 2)	Value	Uncertainty (Note 2)
30	0.982	0.004	0.0688	0.0033	-81.1	2.8
50	0.987	0.004	0.0430	0.0033	-85.3	4.4
100	0.992	0.004	0.0244	0.0034	-90.9	7.9
300	0.994	0.005	0.0155	0.0034	-121.9	12.6
500	0.994	0.005	0.0173	0.0035	-148.9	11.4
1000	0.993	0.005	0.0236	0.0035	+153.1	8.5
2000	0.986	0.005	0.0345	0.0050	+56.4	8.3
3000	0.981	0.005	0.0323	0.0051	-35.6	9.1

**Notes:**

1. The calibration factor,  $K$ , relates the total incident power to the DC power substituted by the self-balancing bridge. It is defined as the ratio of DC substituted power,  $P^{\text{DC}}$ , to total incident RF power  $P^{\text{RF}}$ :

$$K = \frac{P^{\text{DC}}}{P^{\text{RF}}} = (1 - |\Gamma|^2) \eta,$$

where  $\Gamma$  is the input reflection coefficient of the mount and  $\eta$  is the mount effective efficiency.

2. The expanded uncertainties quoted in this report are for a level of confidence of approximately 95%. They were calculated using a coverage factor  $k = 2$ . See the ISO Guide to the Expression of Uncertainty in Measurement (ISO, 1<sup>st</sup> edition, 1995) for an explanation of terms.

### 3.2 NML Measurement Result

The measured calibration factor and reflection coefficient of the travelling thermistor mount are shown in Table 2.

TABLE 2

Frequency (MHz)	Calibration Factor (Note 1)	Uncertainty (Note 2 and 3)	Reflection Coefficient $ \Gamma $	Uncertainty (Note 2)
30	0.985	$\pm 0.40\%$	0.0635	$\pm 0.0063$
50	0.988	$\pm 0.40\%$	0.0397	$\pm 0.0056$
100	0.993	$\pm 0.40\%$	0.0233	$\pm 0.0040$
300	0.994	$\pm 0.40\%$	0.0154	$\pm 0.0047$
500	0.994	$\pm 0.40\%$	0.0170	$\pm 0.0048$
1000	0.993	$\pm 0.42\%$	0.0238	$\pm 0.0049$
2000	0.985	$\pm 0.44\%$	0.0356	$\pm 0.0050$
3000	0.981	$\pm 0.46\%$	0.0360	$\pm 0.0051$

Note 1: The calibration factor is defined as the ratio of the DC power withdrawn by the self balancing bridge associated with the thermistor mount, to the incident RF power.

Note 2: The uncertainty has been calculated in accordance with principles in the ISO Guide to the Expression of Uncertainty in Measurement, and gives an interval estimated to have a level of confidence of 95%. The coverage factor is 2.

Note 3: The uncertainty value in this column is expressed in percentage of measured value of the calibration factor.

## 4 Uncertainty

### 4.1 MSL Uncertainty Budget

The equation used to transfer a value of calibration factor,  $K$ , from a calibrated thermistor mount to an uncalibrated mount at a particular frequency is

$$K_u = K_c \frac{P_u^{\text{DC}}}{P_c^{\text{DC}}} \frac{|1 - \Gamma_s \Gamma_u|^2}{|1 - \Gamma_s \Gamma_c|^2}, \quad (1)$$

where the subscripts ‘u’, ‘c’ and ‘s’ label the uncalibrated mount, calibrated mount and splitter match respectively, and  $P^{\text{DC}}$  refers to the DC substituted power to a mount.

It is convenient to consider separately three factors in this equation:

1. the uncertainty in the standard value of calibration factor:  $K_c$ ;
2. the uncertainty in the power ratio:  $P_u^{\text{DC}}/P_c^{\text{DC}}$  (includes the uncertainty contributed by using reference arm measurements to correct for source instability);
3. the uncertainty in the mismatch correction factor:  $|1 - \Gamma_s \Gamma_u|^2/|1 - \Gamma_s \Gamma_c|^2$ .

Table 3 summarises these uncertainty contributions at the frequencies measured (all uncertainties in the table are standard uncertainties). Degrees-of-freedom are reported for the power ratio only, because the type-A contribution to the power measurement is significant. For the standard  $K_c$  and the mismatch correction, degrees-of-freedom are taken as infinite.

The measurement of calibration factor is traceable through the calibration certificate of the standard mount. In this comparison, the MSL standard had been calibrated at NML.

The uncertainty calculations required for the mismatch correction involve complex-valued quantities, to which the methods of the GUM do not apply. We have used the bivariate extensions to the ‘Law of propagation of uncertainty’ described in [4]. The mismatch correction is a scalar, and these uncertainty calculations can produce a scalar uncertainty statement for this result.

TABLE 3

Frequency (MHz)	Standard $K_c$		Power ratio			Mismatch correction	
	Value	u	Value	u	DoF	Value	u
30	0.983	0.0020	0.9993	0.00017	11	0.99999	$2.9 \times 10^{-5}$
50	0.986	0.0020	1.0014	0.00010	53	1.00001	$3.0 \times 10^{-5}$
100	0.990	0.0020	1.0020	0.00012	31	1.00002	$3.2 \times 10^{-5}$
300	0.991	0.0025	1.0025	0.00011	26	1.00004	$3.8 \times 10^{-5}$
500	0.991	0.0025	1.0028	0.00010	85	1.00006	$4.4 \times 10^{-5}$
1000	0.989	0.0025	1.0035	0.00010	91	1.00015	$6.2 \times 10^{-5}$
2000	0.982	0.0025	1.0037	0.00013	20	1.00037	$16 \times 10^{-5}$
3000	0.977	0.0025	1.0044	0.00011	39	1.00002	$27 \times 10^{-5}$

## 4.2 NML Uncertainty Budget

The output of a synthesiser was connected to port 1 of the power splitter; the DUT and the reference mounts were connected to port 2 and port 3 of the splitter, respectively.

The equation used to calculate calibration factor,  $K_T$ , from the calibration factor  $K_R$  of the reference thermistor mount is

$$K_T = K_R \frac{P_{mT}}{P_{mR}} \frac{|s_{31}|^2}{|s_{21}|^2} \frac{\left| 1 - \left( s_{22} - \frac{s_{21}s_{23}}{s_{31}} \right) \Gamma_T \right|^2}{\left| 1 - \left( s_{33} - \frac{s_{31}s_{23}}{s_{21}} \right) \Gamma_R \right|^2}, \quad (2)$$

where  $P_{mT}$  and  $P_{mR}$  are indicated powers of the test and the reference mounts,  $s_{ij}$  are scattering parameters of the splitter, and  $\Gamma_T$  and  $\Gamma_R$  are reflection coefficients of the test and the reference mount, respectively.

The four main components of the uncertainty and their estimated values are listed in Table 4 below:



TABLE 4

Frequency (MHz)	$u_{\text{ref}}(\%)$	$u_{\text{pm}}(\%)$	$u_{\text{opt}}(\%)$	$u_{\text{misMatch}}(\%)$	$U_{\text{comb}}(\%)$
30	0.132	0.086	0.119	0.003	0.20
50	0.132	0.086	0.119	0.003	0.20
100	0.132	0.086	0.119	0.005	0.20
300	0.132	0.086	0.119	0.007	0.20
500	0.135	0.086	0.125	0.008	0.20
1000	0.141	0.089	0.131	0.021	0.21
2000	0.145	0.092	0.132	0.024	0.22
3000	0.151	0.095	0.147	0.035	0.23

where  $u_{\text{ref}}$  is the uncertainty in the calibration factor  $K_{\text{R}}$  in the reference mount,  $u_{\text{pm}}$  is the uncertainty of the ratio of indicated powers,  $u_{\text{opt}}$  is the uncertainty in the square of the output tracking (third term of equation (2)) of the splitter and  $u_{\text{misMatch}}$  is the equivalent mismatch uncertainty.  $U_{\text{comb}}$  is the combined uncertainty and is the RSS value of the above four components ( $k = 1$ ).

## 5 Relationship to CCEM.RF-K8

The protocol used in this supplementary comparison is based on the protocol for the recent key comparison CCEM.RF-K8, however the frequency points are mostly different (there are only points in common, at 50 MHz and 1 GHz). The range of frequencies used in this comparison (30 MHz – 3 GHz) is contained within the range used in CCEM.RF-K8 (10 MHz – 18 GHz).

The pilot laboratory, NML, took part in CCEM.RF-K8. However, as already noted, MSL obtained traceability for calibration factor measurements through a transfer standard calibrated at NML and the uncertainty in the standard dominates the MSL uncertainty budget. As a consequence, the two laboratories' results will be strongly correlated.

## 6 Conclusions

The measured calibration factors and their associated uncertainties ( $k = 2$ ) from both laboratories are shown in Table 5 below. For comparison, the difference between the two is listed in the last column:

TABLE 5

Frequency (MHz)	NML		MSL		Difference
	Value	Uncertainty	Value	Uncertainty	
30	0.985	$\pm 0.0040$	0.982	$\pm 0.004$	0.003
50	0.988	$\pm 0.0040$	0.987	$\pm 0.004$	0.001
100	0.993	$\pm 0.0040$	0.992	$\pm 0.004$	0.001
300	0.994	$\pm 0.0040$	0.994	$\pm 0.005$	0.000
500	0.994	$\pm 0.0040$	0.994	$\pm 0.005$	0.000
1000	0.993	$\pm 0.0042$	0.993	$\pm 0.005$	0.000
2000	0.985	$\pm 0.0044$	0.986	$\pm 0.005$	0.001
3000	0.981	$\pm 0.0046$	0.981	$\pm 0.005$	0.000

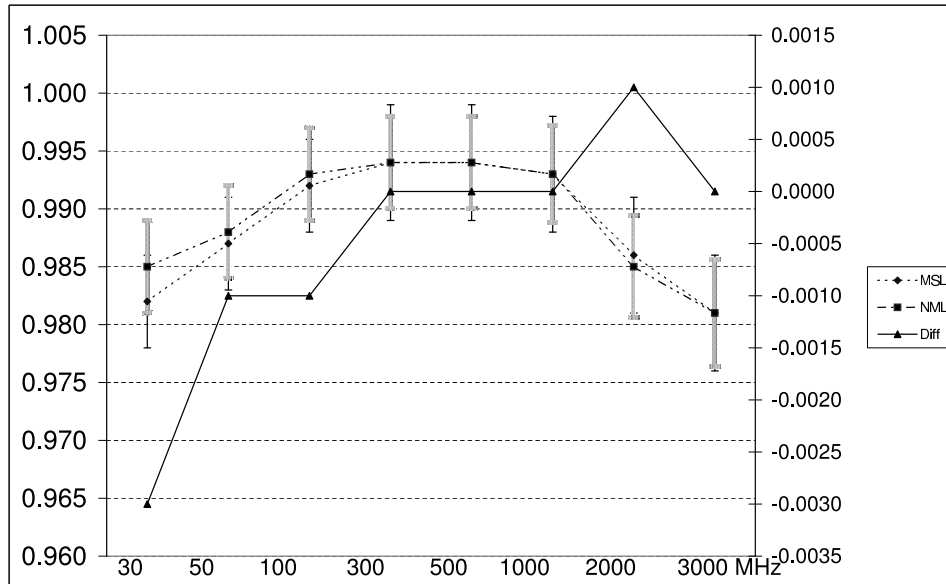


Figure 1: The scale on the left applies to the measured values of calibration factor, which are joined with dotted lines. The scale on the right applies to the differences between these values, which are joined with a solid line.

The measured calibration factors are also plotted in Figure 1, together with the differences between them. The maximum difference is 0.003 at 30 MHz, where the

estimated uncertainty ( $k = 2$ ) for each laboratory is 0.004. The values at other frequencies agree within 0.001. The agreement between the laboratories is therefore judged to be satisfactory.

## References

- [1] N. T. Larsen, “A *new self-balancing DC-substitution RF power meter*”, IEEE Trans. Inst. Meas., **25**(4) (1976) 343-347.
- [2] J. R. Juroshek, “A *direct calibration method for measuring equivalent source mismatch*”, Microwave J., **40**(10) (1997) 106-118.
- [3] T. Zhang, “A *novel approach for power calibrations using power splitters*”, in the Conference Digest of the 24<sup>th</sup> Conference on Precision Electromagnetic Measurements (CPEM2004, London, UK, 27 June to 2 July 2004).
- [4] N. M. Ridler and M. J. Salter, “A *n approach to the treatment of uncertainty in complex S-parameter measurements*” Metrologia, **39** (2002) 295-302.