

**Bilateral Comparison of 10 V Standards
between the NML (Ireland) and the BIPM,
April to May 2009
(part of the ongoing BIPM key comparison BIPM.EM-K11.b)**

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As a part of the ongoing BIPM key comparison BIPM.EM-K11.b, a comparison of the 10 V voltage reference standards of the BIPM and the National Metrology Laboratory (NML), Dublin, Ireland, was carried out from April to May 2009. Two BIPM Zener diode-based travelling standards (Fluke 732B), BIPM_A (SN:7480002) and BIPM_C (SN:7195014), were transported by freight to NML. The NML measurements were carried out by comparison with the mean of the NML voltage standard. The output EMF of each travelling standard, at its 10 V output terminals, was measured by comparison with the NML voltage standard, which is comprised of ten commercially-produced Zener diode-based electronic voltage standards. Each of them has been characterized for the effects of temporal drift and sensitivity to environmental conditions. The value ascribed to the ensemble voltage standard is the weighted mean of the values of the individual standards. A least-squares method is used to reduce the measurement data, which comprise the measured voltage differences and the predicted values of the NML reference standards.

At the BIPM, the traveling standards were calibrated with the Josephson Voltage Standard. Results of all measurements were corrected for the dependence of the output voltages on internal temperature and ambient pressure.

Figure 1 shows the measured values obtained for the two standards by the two laboratories. A linear least squares fit is applied to the results of both laboratories to obtain the results for both standards and their uncertainties at a common reference date.

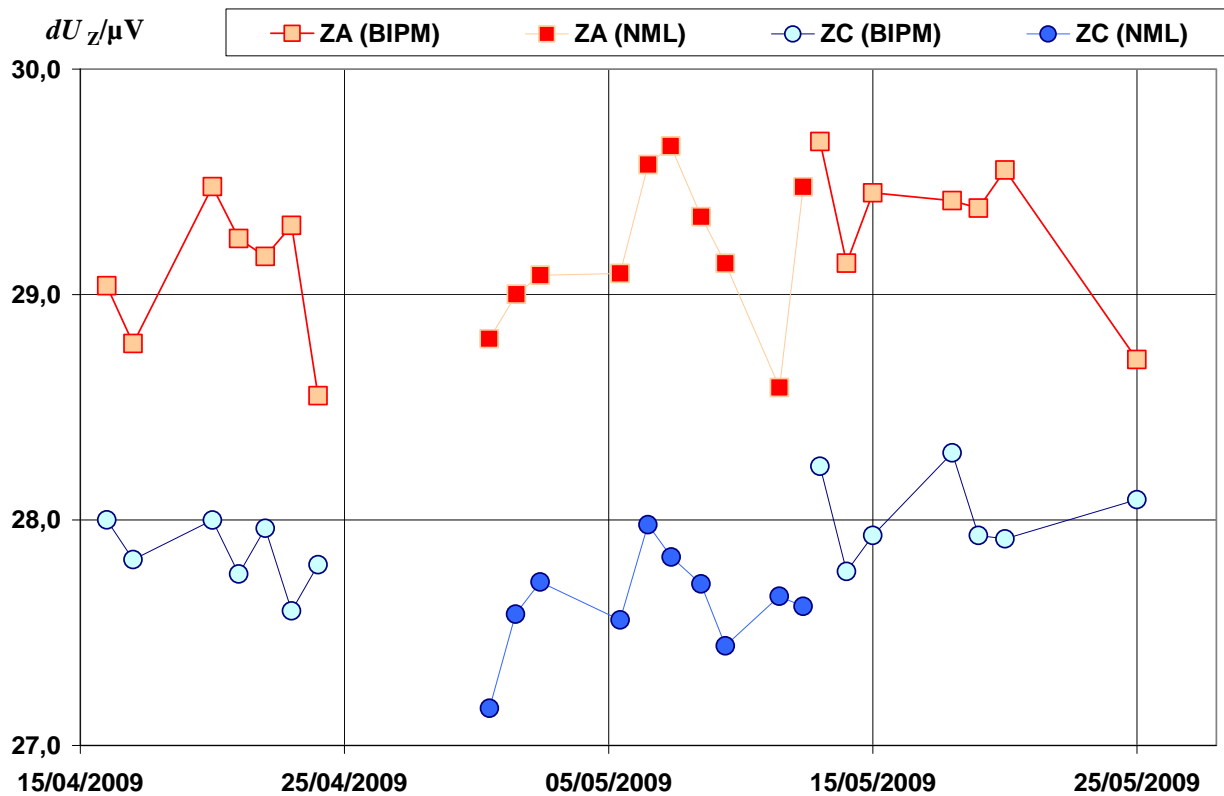


Figure 1. Voltage of BIPM_A (in red) and BIPM_C (in blue) at 10 V measured at both institutes, referred to an arbitrary origin, as a function of time, with linear least-squares fit to the measurements of both laboratories.

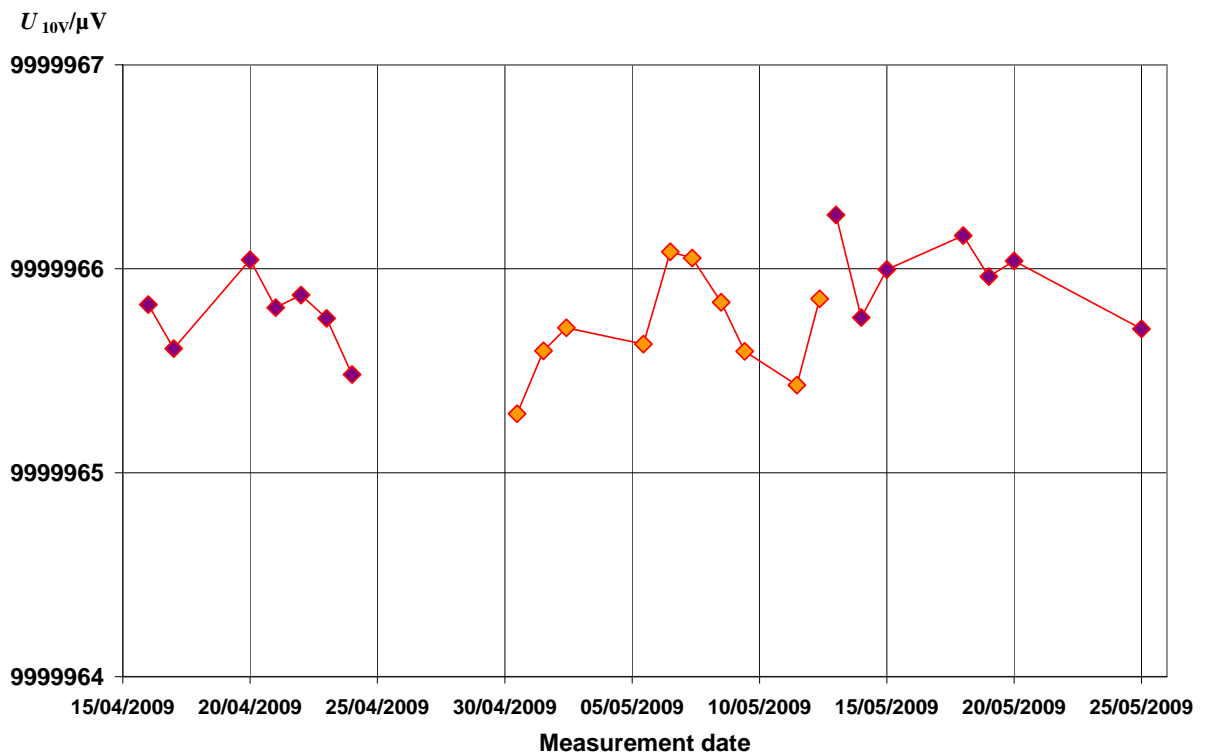


Figure 2. Voltage evolution of the simple mean of the two standards

Table 1 lists the results of the comparison and the uncertainty contributions for the comparison NML/BIPM. Experience has shown that flicker or $1/f$ noise ultimately limits the stability characteristics of Zener diode standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1 part in 10^8 .

In estimating the uncertainty related to the stability of the standards during transportation, we have calculated the *a priori* uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination of the uncorrelated uncertainties of each result. We compared this component to the *a posteriori* uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two traveling standards*. If the *a posteriori* uncertainty is significantly larger than the *a priori* uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

In Table 1, the following elements are listed:

- (1) the predicted value U_{NML} of each Zener, computed using a linear least-squares fit to all of the data from the NML and referenced to the mean date of the NML's measurements;
- (2) the Type A uncertainty due to the instability of the Zener, computed as the standard uncertainty of the value predicted by the linear drift model, or as an estimate of the $1/f$ noise voltage level, whichever is greater;
- (3) the uncertainty component arising from the maintenance of the volt at the NML: this uncertainty is completely correlated between the different Zeners used for a comparison;
- (4-6) the corresponding quantities for the BIPM referenced to the mean date of the NML's measurements;
- (7) the uncertainty due to the combined effects of the uncertainties of the pressure

* With only two traveling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.

and temperature coefficients and to the difference of the mean pressures and temperatures in the participating laboratories is calculated using the following assumption:

An average of the uncertainties of the temperature coefficients of both Zener standards is calculated. The uncertainty on the temperature correction is then considered for the difference between the mean value of the temperature measured at both institutes.

$u_T = U \times u_{CT} \times \Delta R$ where $U = 10 \text{ V}$, $u_{CT} = 4.35 \times 10^{-8} / \text{k}\Omega$, $\Delta R = 0.1 \text{ k}\Omega$ for ZA and $\Delta T = 0.07 \text{ k}\Omega$ for ZC.

The same procedure is applied for the uncertainty on the pressure correction for the difference between the mean value of the pressure measured at both institutes:

$u_P = U \times u_{CP} \times \Delta P$ where $U = 10 \text{ V}$, $u_{CP} = 0.07 \times 10^{-9} / \text{hPa}$, $\Delta P = 3.1 \text{ hPa}$ for ZA and $\Delta P = 3.2 \text{ hPa}$ for ZC.

(8) the difference ($U_{\text{NML}} - U_{\text{BIPM}}$) for each Zener, and (9) the uncorrelated part of the uncertainty;

(10) the result of the comparison is the simple mean of the differences of the calibration results for the different standards;

(11 and 12) the uncertainty related to the transfer, estimated by the following two methods:

(11) the *a priori* uncertainty, determined as described on page 4;

(12) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results;

(13) the correlated part of the uncertainty and

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

Table 2 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM.

Table 3 lists the uncertainties related to the maintenance of the volt and the Zener calibration at the NML.

The final result of the comparison is presented as the difference between the value assigned to a 10 V standard by the NML, at the NML, U_{NML} , and that assigned by the BIPM, at the BIPM, U_{BIPM} , which for the reference date is

$$U_{\text{NML}} - U_{\text{BIPM}} = -0.19 \mu\text{V}; u_c = 1.14 \mu\text{V} \quad \text{on 2009/05/06,}$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at the BIPM, based on $K_{\text{J-90}}$, the uncertainty of the representation of the volt at NML, and the uncertainty related to the comparison.

This is a satisfactory result. The comparison result shows that the voltage standards maintained by the NML and the BIPM were equivalent, within their stated expanded uncertainties, on the mean date of the comparison.

The uncorrelated uncertainty is $w = [r^2 + t^2 + v^2]^{1/2}$, the expected transfer uncertainty (*a priori* uncertainty) is $x = \frac{1}{2} [w_A^2 + w_C^2]^{1/2}$, and the correlated uncertainty is $y = [s^2 + u^2]^{1/2}$, where:

r is the NML Type A uncertainty (2);

s is the NML Type B uncertainty, which is assumed to be correlated for both transfer standards (3);

t is the BIPM Type A uncertainty (5);

u is the BIPM Type B uncertainty, which is assumed to be correlated for both transfer standards (6);

v is the pressure and temperature coefficient correction uncertainty (7);

w_i is the quadratic combination of the uncorrelated uncertainties for the Zener (9);

x is the expected transfer uncertainty (from the calculation of the statistical internal consistency) (11);

y is the quadratic combination of the correlated uncertainties (13).

($U_z - 10V$)

		BIPM_A	BIPM_C	
1	NML (Ireland)($U_z - 10V$)/ μV	-17.32	-51.26	
2	Type A uncertainty/ μV	0.11	0.10	r
3	correlated unc. / μV	1.13		s
4	BIPM ($U_z - 10V$)/ μV	-17.28	-50.94	
5	Type A uncertainty/ μV	0.10	0.10	t
6	correlated unc./ μV	0.01		u
7	pressure and temperature correction uncertainty/ μV	0.04	0.03	v
8	($U_{NML} - U_{BIPM}$)/ μV	-0.05	-0.32	
9	uncorrelated uncertainty/ μV	0.15	0.14	w
10	$\langle U_{NML} - U_{BIPM} \rangle$ / μV	-0.19		
11	<i>a priori</i> uncertainty/ μV	0.103		x
12	<i>a posteriori</i> uncertainty/ μV	0.138		
13	correlated uncertainty/ μV	1.13		y
14	comparison total uncertainty/ μV	1.14		

Table 1. Results of the NML(Ireland)/BIPM bilateral comparison of 10 V standards using two Zener traveling standards: reference date 06 May 2009. Uncertainties are 1σ estimates.

JVS & detector uncertainty components	Uncertainty/nV
Residual thermal electromotive forces	included in the Type A uncertainty
Meas. loop noise/ electromagnetic interference	0.86
detector freedom from bias	0.1
leakage resistance	0.06
frequency	0.08
pressure and temperature correction	included in the Zener unc. budget
total	0.9

Table 2. Estimated standard uncertainties for Zener calibrations with the BIPM equipment at the level of 10 V without the contribution of the Zener noise. The Comparison Type A uncertainty was calculated as the standard deviation of the mean of the BIPM daily measurement results and is equal to 55 nV. But we consider that this component can't be lower than the 1/f noise floor estimated at 100 nV.

	Uncertainty/μV
reference group stability and comparator	1.12
temperature and pressure correction	Included in the Zener unc. budget
Non-repeatability	0.1
total	1.13

Table 3. Estimated standard uncertainties for Zener calibrations with the NML equipment.