BUREAU INTERNATIONAL
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Bilateral Comparison of 1 Ω standards
(ongoing BIPM key comparison BIPM.EM-K13.a)
between the NMIA (Australia) and the BIPM.

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1 Introduction

A comparison of values assigned to 1 Ω resistance standards was carried out between the BIPM and the NMIA (Australia) in the period September 2008 to January 2009.

Three 1 Ω BIPM travelling standards of CSIRO type were calibrated first at the BIPM, then at the NMIA and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: September – October 2008
NMIA measurements: October – November 2008
'After' measurements at the BIPM: December 2008 – January 2009

The BIPM calibrations are corrected to the reference temperature 23.000 °C and the reference pressure 1013.25 hPa.
According to the protocol, the NMIA does not apply pressure and temperature corrections to its results. The corrections are made by the BIPM, using the temperature and pressure coefficients of the standards together with the temperature and pressure measurements provided by the NMIA.

There is no clear evidence of a single linear drift of each standard over the whole period of the comparison (three measurement periods, 'Before', 'NMIA' and 'After': see Figures 1, 2 and 3). In particular, two of them exhibited a step change after their return. During each period, the resistance of each standard is therefore assumed to be constant, with superimposition of a random noise.

For each period, the calibration value assigned to each standard is the mean value of the measurements performed during this period, with an associated standard uncertainty.

The difference between the NMIA and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{NMIA},\,i} - R_{\mathrm{BIPM},\,i}$$

If three standards are used, the mean of the differences is:

$$\Delta_{\mathrm{NMIA-BIPM}} = \frac{1}{3}\sum_{i=1}^{3}(R_{\mathrm{NMIA},\,i} - R_{\mathrm{BIPM},\,i}) \qquad (1)$$

This expression can also be written as:

$$\Delta_{\mathrm{NMIA-BIPM}} = \frac{1}{3}\sum_{i=1}^{3}R_{\mathrm{NMIA},\,i} - \frac{1}{3}\sum_{i=1}^{3}R_{\mathrm{BIPM},\,i} \qquad (2)$$

which is the difference of the means.

2 Measurements at the BIPM

2.1 BIPM calibrations

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor (referred to as BI100-3) whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The

comparison is performed using a DC cryogenic current comparator operating with a 50 mA current in the 1 Ω resistors.
In order to minimize the extrapolation uncertainty, the 100 Ω reference resistor was calibrated against the QHR in September 2008, during the first part of the comparison.
The 1 Ω resistors were kept in a temperature controlled oil bath at a temperature which is close (within a few mK) to the reference temperature. The temperature of the standards is determined by means of a calibrated SPRT, in conjunction with thermocouples.

The BIPM measurements are summarized in Table 1 and the uncertainty budget in Table 2.

BIPM	**Relative difference from nominal 1 Ω value / 10^{-6}**			
Standard #	BEFORE	S. dev. mean u_{1B}	AFTER	S. dev. mean u_{1A}
64200	– 0.784	0.001	– 0.827	0.001
64203	+ 0.465	0.001	+ 0.462	0.001
64207	– 0.553	0.001	– 0.517	0.002

Mean value of 'Before' and 'After'			
Standard #	**mean** / 10^{-6}	Dispersion u_1 / 10^{-9}	Systematic u_2 / 10^{-9}
64200	**– 0.806**	1	16
64203	**+ 0.463**	1	16
64207	**– 0.535**	1	16

Table 1: Summary of the BIPM calibrations. The dispersion is estimated by the standard deviations, and 'systematic' refers to the sources of uncertainty that do not contribute to the variability of the results.

Source of uncertainty	relative standard uncertainty
Imperfect realisation of R_{K-90}	2×10^{-9}
Calibration of the BIPM 100 Ω reference (BI100-3) against R_{K-90}	3×10^{-9}
Interpolation / extrapolation of the value of BI100-3	13×10^{-9}
Measurement of the (1 Ω / BI100-3) ratio	8×10^{-9}
Temperature correction for the 1 Ω standard	2×10^{-9}
Pressure correction for the 1 Ω standard	3×10^{-9}
Combined uncertainty u_2	**16×10^{-9}**

Table 2: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The value attributed to i-th standard is the arithmetic mean of the "Before" and "After" values.

$$R_{\mathrm{BIPM},\,i} = (R_{\mathrm{Before},\,i} + R_{\mathrm{After},\,i}) / 2$$

For each standard, the uncertainty u_1 associated with the dispersion is the quadratic mean of the standard deviations "Before" and "After".

$$u_{1,\,i}^2 = (u_{1\mathrm{Before},\,i}^2 + u_{1\mathrm{After},\,i}^2) / 2^2$$

u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 2. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{\mathrm{BIPM},\,i}$ is obtained from: $u_{\mathrm{BIPM},\,i}^2 = u_{1,\,i}^2 + u_{2,\,i}^2$

The $u_{2,\,i}$ are assumed to be correlated, unlike $u_{1,\,i}$.

Using expression (2), when the mean (for three standards) of the NMIA-BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{\mathrm{BIPM}}^2 = \sum_{i=1}^{3} \frac{u_{1,\,i}^2}{3^2} + u_2^2 \qquad (3)$$

Using the values shown in Table 2, the relative standard uncertainty u_{BIPM} is

$$u_{\mathrm{BIPM}} = 16 \times 10^{-9}.$$

2.2 Uncertainty associated with the transfer

u_{d} is the uncertainty associated with the drift (or the step changes) of the travelling standards observed after their return at the BIPM.

The final resistance value attributed by the BIPM (the mean of the 'Before' and 'After' measurements) is in the middle of the step d: $d = |(R_{\mathrm{After}} - R_{\mathrm{Before}})|$

As we have no clear knowledge about the behaviour of the standards during the period between 'Before' and 'After', it is assumed that the actual resistance could have had any value in the range d, with equal probability.

Assuming a rectangular probability distribution, $u_{\mathrm{d}} = \frac{d}{2} \cdot \frac{1}{\sqrt{3}}$

Another source of uncertainty associated with the transfer is the difference in the operating currents used by the two laboratories: 50 mA at the BIPM and 100 mA at the NMIA. This might influence the resistance of the standards through their power coefficients.

A series of measurements were performed at the BIPM to determine these power coefficients. Unfortunately, our current source does not allow currents significantly larger than 50 mA. The BIPM series of measurements were therefore carried out using alternately I = 50 mA and $I/\sqrt{2}$, but no significant change could be observed within the noise of the measurements, that is about 1 part in 10^9.

A conservative value of the relative standard uncertainty u_{P} associated with possible power effects in the range 50 mA – 100 mA was estimated to be:

$$u_{\mathrm{P}} = 2 \times 10^{-9}.$$

For a single standard, the transfer uncertainty $u_{T,\,i}$ is obtained from: $u_{T,\,i}^2 = u_{d,\,i}^2 + u_{P,\,i}^2$
The $u_{P,\,i}$ are assumed to be correlated, unlike $u_{d,\,i}$.

Following the same reasoning as in expression (3), the uncertainty u_T associated with the transfer (for the mean of three standards) is:

$$u_T^2 = \sum_{i=1}^{3} \frac{u_{d,\,i}^2}{3^2} + u_P^2$$

	TRANSFER	
Standard #	Drift u_d / 10^{-9}	Power u_P / 10^{-9}
64200	12	2
64203	1	2
64207	10	2
Combined	5	2
Total u_T	**6**	

Table 3: Uncertainty associated with the drift and the power coefficient of the standards.

Using the values of Table 3, the relative standard uncertainty u_T is:

$$u_T = 6 \times 10^{-9}$$

3 NMIA results

3.1 Measurement method and traceability

The NMIA quantized Hall resistance system is currently being re-developed. In the interim, Australia's standard of resistance is maintained by reference to the NMIA calculable cross capacitor.

The calculable capacitor realises a capacitance of 1/6 pF traceable to the NMIA primary standard of length. This capacitance is compared with a fixed capacitor of the same value using a transformer ratio substitution bridge. Two further fixed capacitors, also of 1/6 pF, are compared to the first using the same bridge. The three fixed capacitors are then connected in parallel and compared with a 5 pF capacitor using a 10:1 ratio transformer bridge. A build-up to 5 nF is made by successive comparisons of fixed capacitors using the 10:1 bridge.

The 10:1 bridge is then reconfigured to form a quadrature bridge to compare two 5 nF capacitors with two 20 kΩ resistors. The dc value of the parallel combination of the two 20 kΩ resistors is calculated using the ac-dc transfer coefficient known from previous measurement.

The parallel combination of the two 20 kΩ resistors is compared with a Hamon build-up resistor configured to 10 kΩ using a Warshawsky substitution bridge. Finally, the build-up resistor configured to 1 Ω is compared with the comparison resistors, again using the Warshawsky bridge.

3.2 Operating conditions

Temperature: the resistors are kept in oil at a temperature close to 20°C.
Pressure range: 998 hPa to 1012 hPa.
Operating dc current: 100 mA.

3.3 Results

Mean date and time		Mean temperature	Mean pressure	Deviation from nominal value		
				S/N 64200	S/N 64203	S/N 64207
(UTC)		/ °C	/ hPa	/ (μΩ/Ω)	/ (μΩ/Ω)	/ (μΩ/Ω)
31/10/2008	07:52	19.998	998.4	− 0.899	+ 0.366	− 0.626
4/11/2008	08:02	19.998	1009.0	− 0.901	+ 0.354	− 0.629
7/11/2008	08:05	19.998	1002.2	− 0.929	+ 0.326	− 0.666
11/11/2008	08:05	19.998	1012.5	− 0.909	+ 0.339	− 0.652
18/11/2008	07:59	19.998	1005.2	− 0.919	+ 0.342	− 0.653

Table 4: Summary of the NMIA calibrations (corrections for temperature and pressure not applied here).

Source of uncertainty	**Type**	**Standard uncertainty / (μΩ/Ω)**	**Degrees of freedom**
A. Calculable capacitor			
1. Geometrical imperfections	B	0.03	5
2. Gaps between bars	B	0.001	3
3. Gaps between bars and guard bar	B	0.0005	3
4. Eccentricity of guard bar spikes	B	0.007	12
5. Optical alignment	B	0.0003	3
6. Obliquity (telescope aperture)	B	0.005	3
7. Close approach	B	0.001	3
8. Residual gas pressure	B	0.0001	5
9. Laser length standard [1]	B	0.006	infinite
10. Frequency correction	B	0.006	3
11. Residual loading on transformer	B	0.002	3
B. Capacitance build-up			
12. Transformer ratio (four times)	B	0.008	5
13. Loading corrections	B	0.003	5
14. Voltage coefficients [1]	B	0.014	50
15. Bridge balance injection	B	0.002	3
16. Two-port definition	B	0.002	3
C. Quad bridge			
17. Frequency	B	0	-
18. Bridge balance injection	B	0.001	3
19. Two-port definition	B	0.006	3
D. dc measurements			
20. ac-dc resistance transfer	B	0.02	3
21. 10 kΩ comparisons	B	0.002	5
22. 10 kΩ : 1 Ω build-up resistor ratio	B	0.002	5
23. 1 Ω comparisons	B	0.002	5
E. Measurement scatter [2]	A	-	-
F. Long term reproducibility	A	0.029	18
Total uncertainty u_2 =		**0.051**	**26.4**
Estimated 1-σ uncertainty in the mean temperature is 0.002 °C with 40 degrees of freedom. Estimated 1-σ uncertainty in the mean pressure is 0.8 hPa with infinite degrees of freedom.			

Table 5: NMIA uncertainty budget.

1. Uncertainty components 9 and 14 have been revised: all other components are as given in [2].
2. As each individual measurement is reported in Section 3, measurement scatter uncertainty is not included here.

The relative temperature coefficients α_{20} and β were provided by the NMIA, which is also the manufacturer of these resistors. These coefficients are used to calculate $R(20)$ according to the relation:

$$R(20) = R(T) - \alpha_{20}(T-20) - \beta(T-20)^2$$

where $R(T)$ is the resistance measured at the temperature T.

The value $R(23)$ of the resistance corrected for 23 °C is:

$$R(23) = R(20) + \alpha_{20}(23-20) + \beta(23-20)^2$$

The NMIA results are corrected to the reference temperature and the reference pressure using the coefficients α_{20}, β and γ shown in Table 6.

	Relative temperature coefficients		Relative pressure coefficients.
Standard #	α_{20} / (10^{-6}/K)	β / (10^{-6}/K²)	γ / (10^{-9}/hPa)
64200	– 0.0046	– 0.0004	– 0.10
64203	+ 0.0002	– 0.0016	– 0.20
64207	– 0.0096	+ 0.0001	– 0.04

Table 6: Temperature and pressure coefficients of the travelling standards.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa		
	Relative corrections applied to the NMIA results	
Standard #	For temperature	For pressure
64200	– 17 x 10^{-9}	– 1 x 10^{-9}
64203	– 14 x 10^{-9}	– 2 x 10^{-9}
64207	– 28 x 10^{-9}	0 x 10^{-9}

Table 7: Corrections for temperature and pressure applied to the NMIA results.

The uncertainty associated with the small corrections for pressure is assumed to be negligible.
The NMIA uncertainty on temperature measurements is small (0.002°C). The uncertainty associated with the large correction for temperature (from 20°C to 23°C) is therefore dominated by the uncertainty on the temperature coefficients. A conservative value of the standard uncertainty u_3 associated with temperature and pressure corrections was estimated to be:

$$u_3 = 0.005 \times 10^{-6}$$

In Table 8, u_1 is the uncertainty associated with the dispersion (experimental standard deviation of the mean), u_2 the measurement uncertainty stated by the NMIA (see Table 5) and u_3 the uncertainty associated with the corrections for temperature and pressure made by the BIPM.

NMIA After corrections	Relative difference from nominal value / 10^{-6}	Relative standard uncertainties		
		dispersion u_1 / 10^{-9}	NMIA budget u_2 / 10^{-9}	corrections u_3 / 10^{-9}
64200	– 0.930	0.006	0.051	0.005
64203	+ 0.330	0.007	0.051	0.005
64207	– 0.674	0.008	0.051	0.005

Table 8: Summary of the NMIA results, after corrections for temperature and pressure.

For a single standard, the NMIA uncertainty $u_{\mathrm{NMIA},\,i}$ is obtained from: $u^2_{\mathrm{NMIA},\,i} = u^2_{1,\,i} + u^2_{2,\,i} + u^2_{3,\,i}$

The $u_{2,\,i}$ and $u_{3,\,i}$ are assumed to be correlated. The $u_{1,\,i}$ are assumed to be uncorrelated (although some correlation, produced by the measurement chain, can be seen on the graphs).

Using expression (2), when the mean (for three standards) of the NMIA-BIPM relative difference is calculated, the NMIA contribution to the uncertainty is:

$$u^2_{\mathrm{NMIA}} = \sum_{i=1}^{3} \frac{u^2_{1,\,i}}{3^2} + u^2_2 + u^2_3 \tag{5}$$

Using the values shown in Table 8 the relative standard uncertainty u_{NMIA} is mainly dominated by u_2:

$$u_{\mathrm{NMIA}} = 0.0514 \times 10^{-6}.$$

4 Comparison NMIA – BIPM

The differences between the values assigned by the NMIA at the NMIA, R_{NMIA}, and those assigned by the BIPM at the BIPM, R_{BIPM}, to each of the three travelling standards during the period of the comparison are shown in Table 9.

NMIA - BIPM	
Standard #	$(R_{\mathrm{NMIA}} - R_{\mathrm{BIPM}})$ / (1 Ω) / 10^{-6}
64200	– 0.124
64203	– 0.133
64207	– 0.139
mean	**– 0.132**

Table 9: Differences between the values assigned by the NMIA (R_{NMIA}) and by the BIPM (R_{BIPM}) to the three travelling standards.

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u^2_{\mathrm{C}} = u^2_{\mathrm{BIPM}} + u^2_{\mathrm{NMIA}} + u^2_{\mathrm{T}}$$

where $u_{\mathrm{BIPM}} = 0.016 \times 10^{-6}$,
$u_{\mathrm{NMIA}} = 0.051 \times 10^{-6}$,
$u_{\mathrm{T}} = 0.006 \times 10^{-6}$
as calculated in sections 2 and 3: $u_{\mathrm{C}} = 0.054 \times 10^{-6}$

A first presentation of the mean difference between the NMIA and the BIPM calibrations, associated with its relative expanded uncertainty U_{C} (expansion factor $k = 2$, 95% confidence level) is:

$$(R_{\mathrm{NMIA}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = -\ 0.132 \times 10^{-6} \tag{6}$$

$$U_{\mathrm{C}} = 0.11 \times 10^{-6}$$

4.1 Choice of value for R_{K}

As is conventional for measurements traceable to the quantum Hall effect (QHE), the BIPM values are based on the standard value $R_{\mathrm{K\text{-}90}}$ = 25 812.807 Ω for the von Klitzing constant R_{K}.
The NMIA values are traceable to a calculable capacitor, which is a direct realisation of the farad. Following the recommendation of the CIPM and CCEM [3], a standard uncertainty of 1×10^{-7} applies to

the use of the QHE as a representation of the ohm. This extra uncertainty should be applied in the present case of a comparison of calculable capacitor and QHE based measurements. The uncertainty of the comparison thus becomes:

$$u_{\mathrm{C}}^2 = u_{\mathrm{BIPM}}^2 + u_{\mathrm{NMIA}}^2 + u_{\mathrm{T}}^2 + u_{R_k}^2$$

where $u_{\mathrm{BIPM}} = 0.016 \times 10^{-6}$,
$u_{\mathrm{NMIA}} = 0.051 \times 10^{-6}$,
$u_{\mathrm{T}} = 0.006 \times 10^{-6}$
$u_{R_{\mathrm{K}}} = 0.1 \times 10^{-6}$
That is: $u_{\mathrm{C}} = 0.114 \times 10^{-6}$

In that case, the presentation of the mean difference between the NMIA and the BIPM calibrations associated with its relative expanded uncertainty U_{C} (expansion factor $k = 2$, 95% confidence level) is:

$$(R_{\mathrm{NMIA}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = -\ 0.132 \times 10^{-6} \qquad (7)$$

$$U_{\mathrm{C}} = 0.23 \times 10^{-6}$$

An alternative calculation of the uncertainty is to use the latest available value for R_{K}, and to omit the above recommended uncertainty. At the time of writing of this report, the best available value for R_{K} is that from the 2006 CODATA adjustment of the constants [4], namely $R_{\mathrm{K}} = 25\ 812.807\ 557(18)\ \Omega$. The relative uncertainty on this value is 6.8×10^{-10}, and the relative difference from $R_{\mathrm{K-90}}$ is

$$(R_{\mathrm{K}} - R_{\mathrm{K-90}}) / R_{\mathrm{K-90}} = +\ 2.1 \times 10^{-8}.$$

Changing the basis of the BIPM measurements to use this value and uncertainty gives the following comparison result (with $k = 2$):

$$(R_{\mathrm{NMIA}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = -\ 0.153 \times 10^{-6} \qquad (8)$$

$$U_{\mathrm{C}} = 0.11 \times 10^{-6}$$

4.1 <u>Final comparison result</u>

In the final result of the comparison, the BIPM results are based on the conventional value $R_{\mathrm{K-90}}$. Those from the NMIA are traceable to a calculable capacitor. The uncertainty of the comparison includes therefore the standard uncertainty of 1×10^{-7} that applies to the use of the QHE as a representation of the ohm.

The result of the comparison is presented as the degree of equivalence D between the NMIA and the BIPM for values assigned to 1 resistance standards, and its expanded relative uncertainty U_{C} (expansion factor $k = 2$, corresponding to a confidence level of 95%),

$$\boldsymbol{D = [(R_{\mathrm{NMIA}} - R_{\mathrm{BIPM}}) / 1\ \Omega] = -\ 0.132 \times 10^{-6}}$$

$$\boldsymbol{U_{\mathrm{C}} = 0.23 \times 10^{-6}}$$

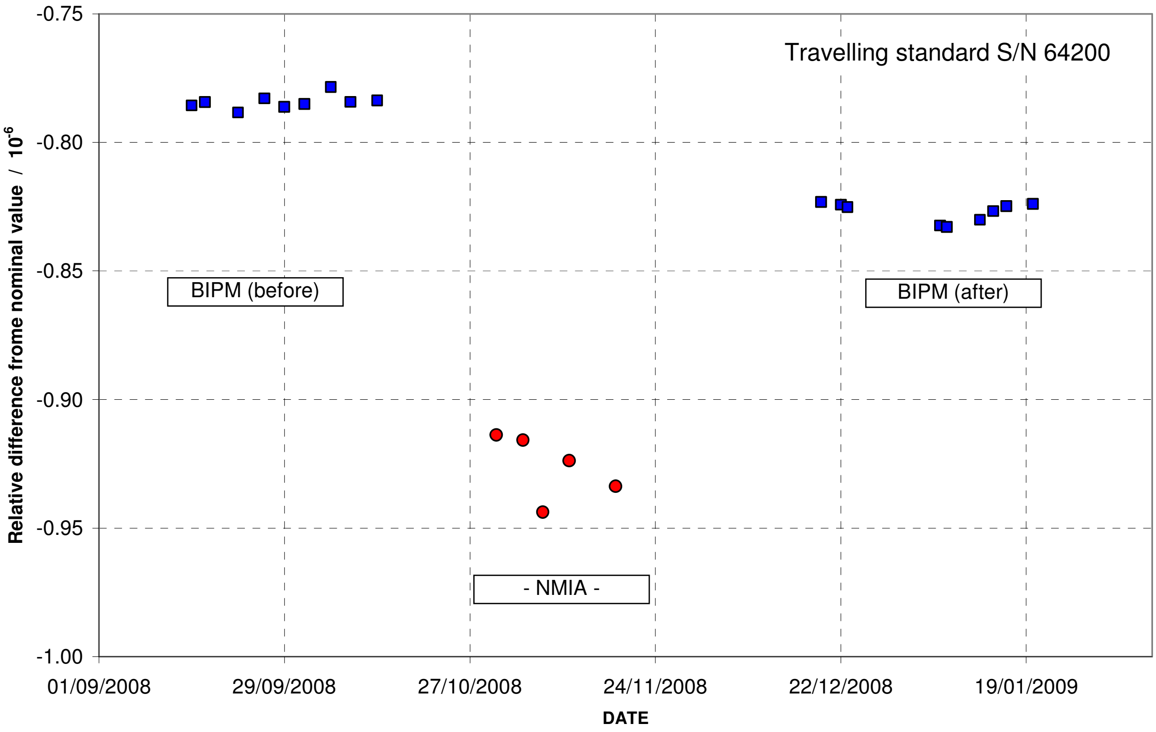

Figure 1: Calibrations at the BIPM (squares) and at the NMIA (circles) of the travelling standard S/N 64200, expressed as the relative deviation from the nominal 1 Ω value.

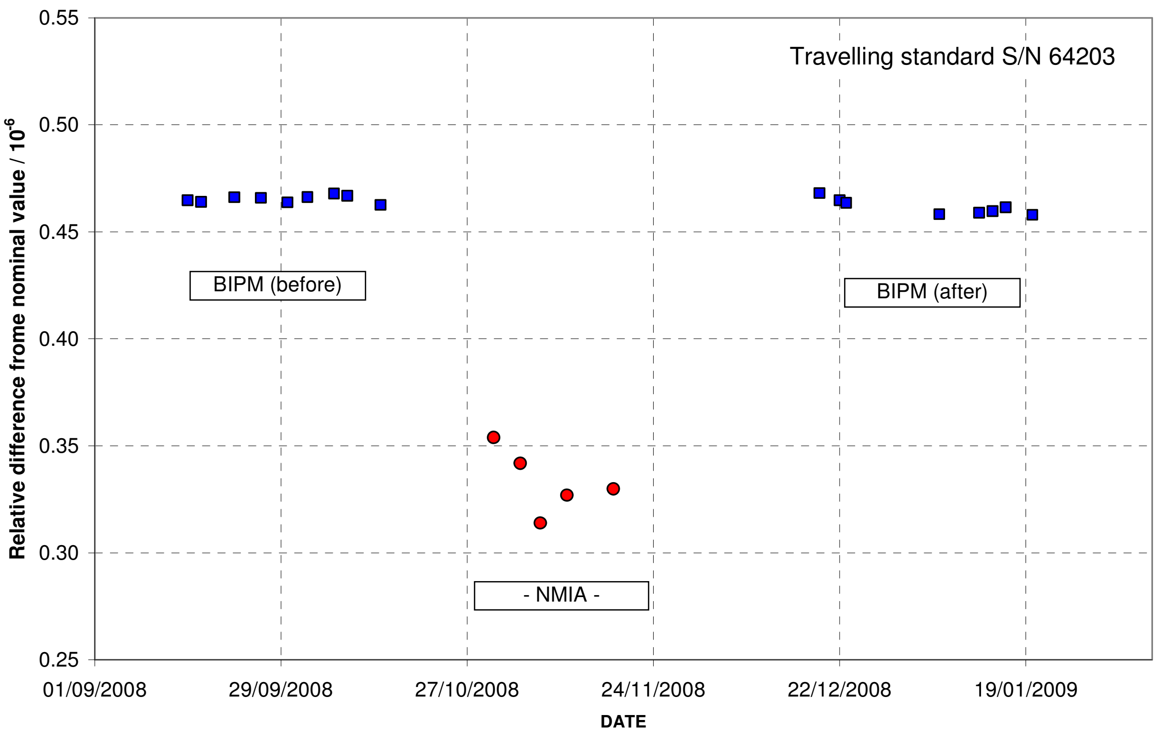

Figure 2: Calibrations at the BIPM (squares) and at the NMIA (circles) of the travelling standard S/N 64203, expressed as the relative deviation from the nominal 1 Ω value.

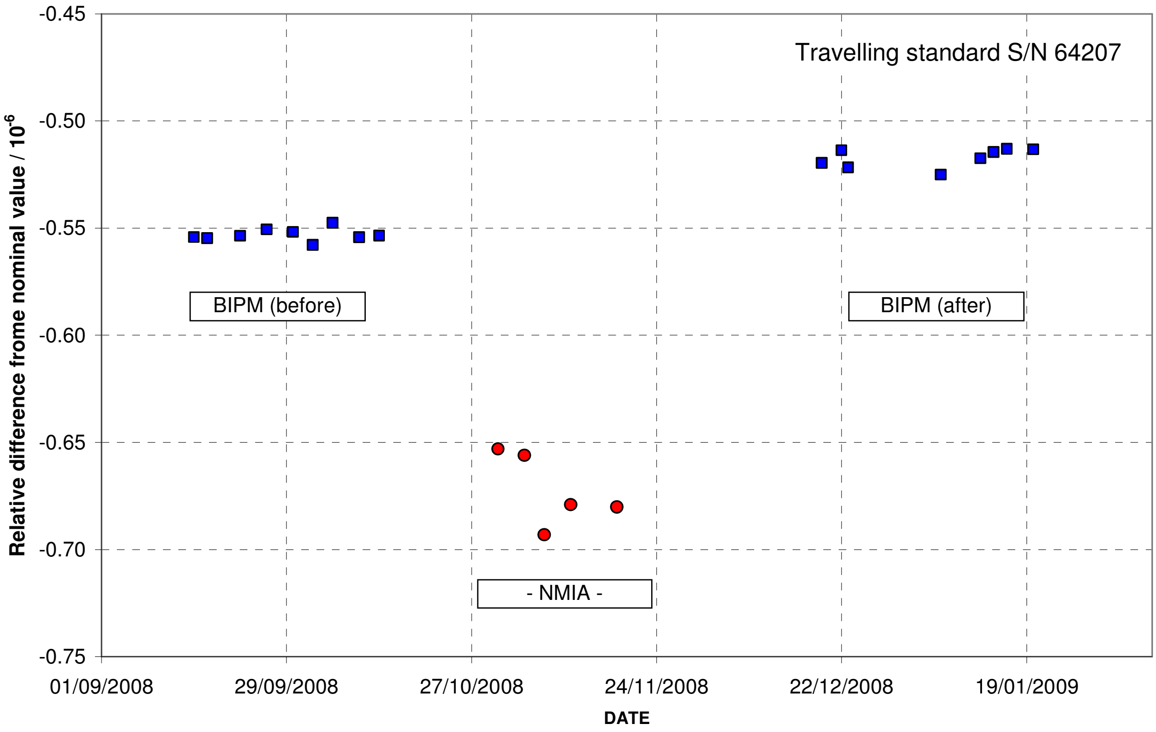

Figure 3: Calibrations at the BIPM (squares) and at the NMIA (circles) of the travelling standard S/N 64207, expressed as the relative deviation from the nominal 1 Ω value.

References:

[1] "Twenty years of SI Ohm Determinations at NML", G W Small, IEEE Trans Instrum & Meas **IM-36**, pp190-195, 1987.

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or: Rev. Mod. Phys. 80, 633-730.