BUREAU INTERNATIONAL
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Bilateral comparison of 1 Ω standards
(ongoing BIPM key comparison BIPM.EM-K13.a)
between the NML (Ireland) and the BIPM

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1 Introduction

A comparison of values assigned to 1 Ω resistance standards was carried out between the BIPM and the NML-Ireland (NMLI) in the period September 2008 to December 2008.

Two 1 Ω BIPM travelling standards of CSIRO type were calibrated first at the BIPM, then at the NMLI and again at the BIPM after their return. The measurement periods are referred to as:
'Before' measurements at the BIPM: September-October 2008
NMLI measurements: October-November 2008
'After' measurements at the BIPM: November-December 2008

The BIPM calibrations are corrected to the reference temperature 23.000 °C and the reference pressure 1013.25 hPa.
According to the protocol, the NMLI did not apply pressure and temperature corrections to its results. The corrections were made by the BIPM, using the temperature and pressure coefficients of the standards together with the temperature and pressure measurements provided by the NMLI.
The calibration reports provided by the NMLI are summarized by the BIPM in section 3 of the present report.

There is no clear evidence of a single linear drift of each standard (see in particular Figure 2) over the whole period of the comparison (three measurement periods, 'Before', 'NMLI' and 'After': see Figures 1 and 2). During each period, the resistance of each standard is therefore assumed to be constant, with superimposition of a random noise.

For each period, the calibration value assigned to each standard is the mean value of the measurements performed during this period, with an associated standard uncertainty.

The difference between the NMLI and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{NMLI},\,i} - R_{\mathrm{BIPM},\,i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\mathrm{NMLI-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}(R_{\mathrm{NMLI},\,i} - R_{\mathrm{BIPM},\,i}) \qquad (1)$$

This expression can also be written as:

$$\Delta_{\mathrm{NMLI-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{NMLI},\,i} - \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{BIPM},\,i} \qquad (2)$$

which is the difference of the means.

The reference standards of the two participants are closely correlated, as the NMLI takes its traceability from the BIPM. The effect of this correlation is reduced by the length of time lapse since the last comparison of NMLI's standards with those of the BIPM, in April 2006.

2 Measurements at the BIPM

2.1 BIPM calibrations

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The comparison was performed using a DC cryogenic current comparator operating with a 50 mA current in the 1 Ω resistors.
The BIPM 100 Ω reference resistor was calibrated against the QHR in September 2008.

The 1 Ω resistors were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature. The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples.

The BIPM measurements are summarized in Table 1 and the uncertainty budget in Table 2.

BIPM	Relative difference from nominal 1 Ω value / 10^{-6}			
Standard #	BEFORE	St. d. mean u_{1B}	AFTER	St. d. mean u_{1A}
64193	– 5.188	0.001	– 5.195	0.001
64205	+ 0.968	0.001	+ 0.916	0.002
Mean value of 'Before' and 'After'				
Standard #	mean / 10^{-6}	Exp. Std. dev. u_1 / 10^{-9}	Systematic u_2 / 10^{-9}	
64193	– 5.191	1	16	
64205	+ 0.942	1	16	

Table 1: Summary of the BIPM calibrations. The dispersion is estimated by the standard deviations, and 'systematic' refers to the sources of uncertainty that do not contribute to the variability of the results.

Source of uncertainty	relative standard uncertainty
Imperfect realization of R_{K-90}	2 x 10^{-9}
Calibration of the BIPM 100 Ω reference against R_{K-90}	3 x 10^{-9}
Interpolation / extrapolation of the value of BI100-3	13 x 10^{-9}
Measurement of the (1 Ω / BI100-3) ratio	8 x 10^{-9}
Temperature correction for the 1 Ω standard	2 x 10^{-9}
Pressure correction for the 1 Ω standard	3 x 10^{-9}
Combined uncertainty u_2	16 x 10^{-9}

Table 2: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The value attributed to the *i*-th standard is the arithmetic mean of the "Before" and "After" values:

$$R_{\text{BIPM, i}} = (R_{\text{Before}, i} + R_{\text{After, i}})/2$$

For each standard, the uncertainty u_1 associated with the dispersion is the quadratic mean of the standard deviations "Before" and "After":

$$u^2_{1,i} = (u^2_{1\text{Before}, i} + u^2_{1\text{After}, i})/2^2$$

u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 2. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{\text{BIPM}, i}$ is obtained from: $u^2_{\text{BIPM}, i} = u^2_{1,i} + u^2_{2,i}$

Unlike $u_{1,i}$, the $u_{2,i}$ are assumed to be correlated.

Using expression (2), when the mean (for two standards) of the NMLI-BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{\mathrm{BIPM}}^2 = \sum_{i=1}^{2} \frac{u_{1,i}^2}{2^2} + u_2^2 \qquad (3)$$

Using the values shown in Table 1, the relative standard uncertainty u_{BIPM} is

$$u_{\mathrm{BIPM}} = 16 \times 10^{-9}.$$

2.2 Uncertainty associated with the transfer

u_{d} is the uncertainty associated with any uncompensated drift or step changes in the values of the travelling standards, as observed by the BIPM.

The final resistance value attributed by the BIPM is the arithmetic mean of the 'Before' and 'After' measurements.

As we have no clear knowledge about the behaviour of the standards during the period between the BIPM 'Before' and 'After' measurements, the value assigned by the BIPM to each standard, on the mean date of the comparison, is taken to lie, with equal probability, in an interval of width $d = \left|\left(R_{After} - R_{Before}\right)\right|$ centred on the mean value.

Assuming a rectangular probability distribution, $u_{\mathrm{d}} = \frac{d}{2} \cdot \frac{1}{\sqrt{3}}$

Another source of uncertainty associated with the transfer would be a difference in the operating currents used by the two laboratories, influencing the resistance of the standards through their power coefficients. In the present case, the nominal operating currents are identical (50 mA) in the two laboratories.

The standards being used with the same current under similar conditions (oil-bath at 23 °C), the influence of their power coefficient is supposed to be similar in both laboratories. The value of the relative standard uncertainty u_{P} associated with possible power effects is therefore estimated to be negligible.

For a single standard, the transfer uncertainty $u_{\mathrm{T},i}$ is obtained from: $u_{\mathrm{T},i}^2 = u_{\mathrm{d},i}^2 + u_{\mathrm{P},i}^2$

The $u_{\mathrm{d},i}$ are assumed to be uncorrelated.

Following the same reasoning as in expression (3), the uncertainty u_{T} associated with the transfer (for the mean of two standards) is:

$$u_{\mathrm{T}}^2 = \sum_{i=1}^{2} \frac{u_{\mathrm{d},i}^2}{2^2}$$

	Transfer	
Standard #	Drift $u_{\mathrm{d}} / 10^{-9}$	Power $u_{\mathrm{P}} / 10^{-9}$
64193	2	0.0
64205	15	0.0
Combined	8	0.0
Total u_{T}	8×10^{-9}	

Table 3: Uncertainty associated with the drift and the power coefficient of the standards.

Using the values of Table 3, the relative standard uncertainty u_{T} is:

$$u_{\mathrm{T}} = 8 \times 10^{-9}$$

3 Measurements at the NMLI

3.1 Measuring method:

The resistance of each standard was measured by comparison with the NMLI 1 Ω reference standard. This standard comprises a group of seven 1 Ω standard resistors whose drift rates are known. The value ascribed to the NMLI standard is the weighted mean of the group. The comparison of the travelling standards with the reference group was carried out using a substitution measuring technique. A direct current comparator resistance bridge (MIL Model 6010C) was used as a transfer standard.

3.2 Operating conditions:

The travelling standards were placed in an oil-bath thermo-regulated at 23 °C and allowed to stabilize.
The actual temperature of the standards and the barometric pressure (including mineral oil) were recorded during each individual calibration.
Operating current: 50 mA dc.
Barometric pressure range: 990 hPa – 1021 hPa.

3.3 NMLI results:

The standards were measured 17 times in the period 30 October – 24 November 2008.
The results are summarized in Table 4.

Serial No. of standard	Mean date of measurement	Resistance value (R_x – 1Ω) / μΩ	Mean temperature / °C	Mean barometric pressure / hPa	Experimental std.dev. mean u_1 / 10^{-6}	Systematic u_2 / 10^{-6}
64193	5 Nov 2008	– 5.16	23.003	1008	0.005	0.051
64205	5 Nov 2008	+ 1.04	23.003	1008	0.004	0.051

Table 4: Summary of the NMLI calibrations. The relative standard uncertainty u_1 refers to the experimental standard deviation of the mean, and u_2 to the uncertainties listed in Table 6.

The NMLI results are corrected to the reference temperature and the reference pressure using the coefficients shown in Table 5.

	Relative temperature coefficients		Relative pressure coefficients.
Standard #	Alpha $_{23}$ / (10^{-6}/K)	Beta / (10^{-6}/K²)	/ (10^{-9}/hPa)
64193	– 0.004	– 0.001	– 0.17
64205	– 0.002	+ 0.002	– 0.2

Table 5: Temperature and pressure coefficients of the travelling standards.

Owing to the small temperature and pressure coefficients of the travelling standards, the corrections for temperature and pressure are well below 1 part in 10^9 in relative terms.

The uncertainty u_3 associated with temperature and pressure corrections is negligible, when compared with the other sources of uncertainty.

Source of uncertainty	Relative standard uncertainty / 10^{-6}
NMLI 1 Ω reference group R_s	0.045
Ratio measurement R_x to R_s	0.026
Correction for temperature (applied by the BIPM)	
Combined (sum in quadrature): u_2	0.0515

Table 6: Summary of the NMLI uncertainty budget (simplified model)

For a single standard, the NMLI uncertainty $u_{\mathrm{NMLI},\,i}$ is obtained from: $u^2_{NMLI,\,i} = u^2_{1,\,i} + u^2_{2,\,i}$
Unlike $u_{1,\,i}$, the $u_{2,\,i}$ are assumed to be correlated.

Using expression (2), when the mean (for two standards) of the NMLI-BIPM relative difference is calculated, the NMLI contribution to the uncertainty is:

$$u^2_{\mathrm{NMLI}} = \sum_{i=1}^{2} \frac{u^2_{1,\,i}}{2^2} + u^2_2 \qquad (5)$$

Using the values shown in Table 4, the relative standard uncertainty u_{NMLI} is

$$u_{\mathrm{NMLI}} = 0.052 \times 10^{-6}.$$

4 Comparison NMLI – BIPM

4.1 Data reduction using the arithmetic mean:

The differences between the values assigned by the NMLI at the NMLI, R_{NMLI}, and those assigned by the BIPM at the BIPM, R_{BIPM}, to each of the two travelling standards during the period of the comparison are shown in Table 7.

Standard #	$\Delta_i = (R_{\mathrm{NMLI}} - R_{\mathrm{BIPM}}) / (1\ \Omega)$ / 10^{-6}
64193	0.035
64205	0.100
mean	0.068

Table 7: Differences between the values assigned by the NMLI (R_{NMLI}) and by the BIPM (R_{BIPM}) to the two travelling standards.

The mean difference between the NMLI and the BIPM calibrations is:

$$(R_{\mathrm{NMLI}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = +\,0.068 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_C, is:

$$u^2_{\mathrm{C}} = u^2_{\mathrm{BIPM}} + u^2_{\mathrm{NMLI}} + u^2_{\mathrm{T}} \qquad (6)$$

where u_{BIPM} = 0.016×10^{-6},
u_{NMLI} = 0.052×10^{-6},
u_{T} = 0.008×10^{-6}
as calculated in sections 2 and 3: $u_{\mathrm{C}} = 0.055 \times 10^{-6}$

4.2 Data reduction using a weighted mean:

In Section 4.1, the two differences Δ_1 and Δ_2 have the same weight in the calculation of the mean, and their uncertainties (slightly different due to different transfer uncertainties) are combined according to the classical expressions (3) and (6).

In another approach, more confidence can be given to a result obtained from a standard showing a better stability (with a lower transfer uncertainty), so that the weights attributed to Δ_1 and Δ_2 would be different.

More generally, Δ_1 and Δ_2 are strongly (but not completely) correlated quantities.
Their mean can be calculated using the generalized weighted mean of correlated quantities [(1)], described below:

$\overline{X} = \sigma_{\overline{x}}^2 \left(W^T C^{-1} X \right)$ is the weighted mean,
where:
$\sigma_{\overline{x}}^2 = \left(W^T C^{-1} W \right)^{-1}$ is the associated variance,

$W = \begin{pmatrix} 1 \\ : \\ 1 \end{pmatrix}$ is the design matrix

C is the covariance matrix

$X = \begin{pmatrix} x_1 \\ : \\ x_n \end{pmatrix}$ the series of data

In order to calculate the covariance matrix, the contributions to the uncertainties associated with Δ_1 and Δ_2, that is $u(\Delta_1)$ and $u(\Delta_2)$ respectively, are grouped in Table 8.
Those marked with (**) are assumed to be fully correlated between $u(\Delta_1)$ and $u(\Delta_2)$.
The uncertainties associated with the dispersion of the measurements in each laboratory (Type A evaluation) and with transport (Type B evaluation of a random effect) are assumed to be uncorrelated between $u(\Delta_1)$ and $u(\Delta_2)$.

Uncertainties Type A, B	Δ_1 / 10^{-6}	Δ_2 / 10^{-6}
u_1 BIPM A	0.001	0.001
u_2 BIPM B (**)	0.016	0.016
u_1 NMLI A	0.005	0.005
u_2 NMLI B (**)	0.051	0.051
u_{Tranfer} B	0.002	0.015
Combined (quadratic sum) :		
All components: $u(\Delta_i)$	0.05373	0.05575
Correlated (**) components $u_{\mathrm{Corr.}}(\Delta_i)$	0.05345	0.05345

Table 8: Contributions to the uncertainties associated with Δ_1 and Δ_2
(numerical values are taken from Tables 1, 3 and 4).

The covariance matrix is then written as:

$$C = \begin{pmatrix} u(\Delta_1)^2 & u_{Corr.}^2 \\ u_{Corr.}^2 & u(\Delta_2)^2 \end{pmatrix} ; \qquad \text{and } W = \begin{pmatrix} 1 \\ 1 \end{pmatrix} ; \qquad X = \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} ;$$

with $\Delta_1 = 0.035 \times 10^{-6}$
$\Delta_2 = 0.100 \times 10^{-6}$

Using the values shown in Table 8, the weighted mean and the associated standard deviation are:

$\overline{X} = 0.042 \times 10^{-6}$
$\sigma = 0.054 \times 10^{-6}$

that is:

$$(R_{\text{NMLI}} - R_{\text{BIPM}}) / (1\ \Omega) = +\ 0.042 \times 10^{-6}$$

$$u_{\text{C}} = 0.054 \text{ x } 10^{-6}$$

As expected, more weight was given to Δ_1 which is associated with a smaller transfer uncertainty.

After discussion between the BIPM and the NMLI, this calculation method was chosen to express the final result.

4.3 Conclusion:

The final result of the comparison is presented as the degree of equivalence D between the NML-Ireland and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor k = 2, corresponding to a confidence level of 95 %) , U_{C}

$$\boldsymbol{D = (R_{\text{NMLI}} - R_{\text{BIPM}}) / 1\ \Omega = +\ 0.042 \times 10^{-6}}$$

$$\boldsymbol{U_{\text{C}} = 0.11 \times 10^{-6}}$$

The NML-Ireland and the BIPM calibrations are in agreement, with a difference smaller than the standard uncertainty.

References:

[1] The generalized weighted mean of correlated quantities, M.G. Cox et al., Metrologia **43** (2006), S268-S275.

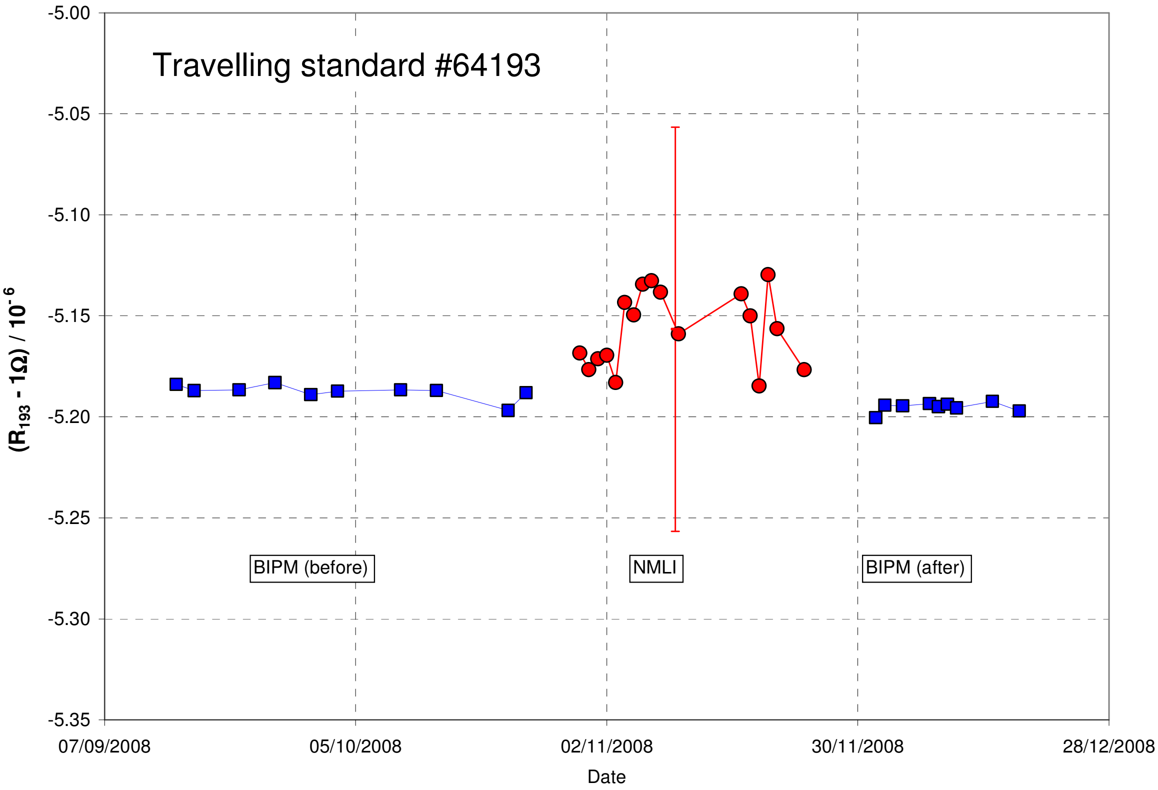

Figure 1: Calibrations at the BIPM (squares) and at the NMLI (circles) of the travelling standard ref. 64193, expressed as the relative deviation from the nominal 1 Ω value. The uncertainty bar corresponds to the NMLI expanded uncertainty (k = 2).

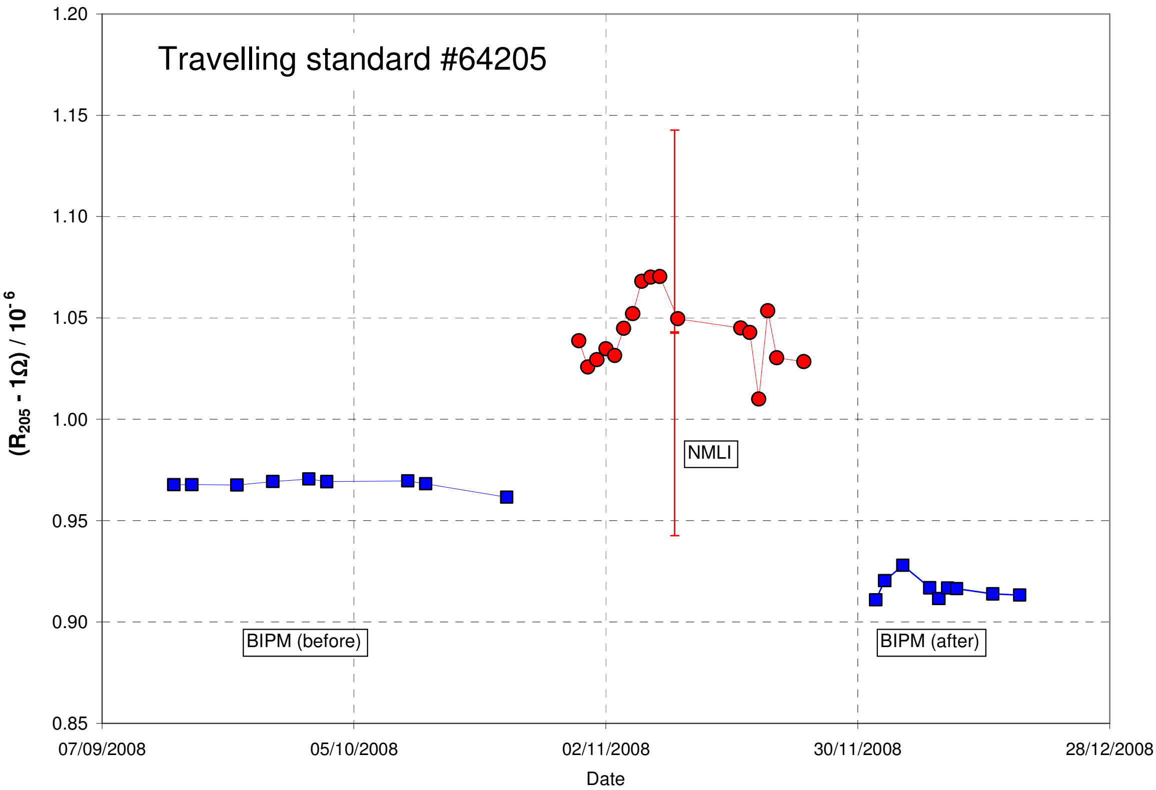

Figure 2: Calibrations at the BIPM (squares) and at the NMLI (circles) of the travelling standard ref. 64205, expressed as the relative deviation from the nominal 1 Ω value. The uncertainty bar corresponds to the NMLI expanded uncertainty (k = 2).