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Citation: [Review of Scientific Instruments](#) **70**, 4715 (1999); doi: 10.1063/1.1150136

View online: <http://dx.doi.org/10.1063/1.1150136>

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Noise investigation of a high precision digital multimeter model HP3457A or HP3458A for microvolt level dc voltage measurements

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(Received 19 January 1999; accepted for publication 15 September 1999)

This article examines noise characteristics of a high precision digital multimeter HP3457A or HP3458A by presenting a conceptual but useful model for the generation of zero voltage error under short-circuited input terminals. Two setting states, auto-zero on (AZ-ON) and off (AZ-OFF) are tested and one important conclusion derived is that the state of AZ-OFF has a more extensive potential of performing to the full specifications for measurements of μV level dc voltage than that of AZ-ON. © 1999 American Institute of Physics. [S0034-6748(99)04212-4]

I. INTRODUCTION

Today, digital multimeters (DMMs), particularly model types HP3457A or HP3458A, have been widely utilized as highly qualified for accurate and precise measurements in various fields.¹⁻⁶ In practice, we have become convinced that they have performed to their specifications by evaluating performances of a faint-leak, current sensor,⁷ of a core noise measurement system,⁸ and of a highly stable thermostat.⁹ However, when we began to make repetition measurements of a dc voltage of μV order or less, using HP3457As (and subsequently, 3458As) on the minimum ranges (0.03 or 0.3 V), we encountered instability in each reading, a nonrepeatability. The transitions cannot be regarded as any thermally generated offset voltage as pointed out in the manual for the HP3457A (at p.3-6) or 3458A (at p.D-4) but irregularly takes a different value at every measurement, even if ambient conditions including room temperature are kept constant in as calm a state as possible. Two settings, auto-zero on (AZ-ON) and off (AZ-OFF), were tested. The reading distribution at AZ-ON spreads over several μV , while at AZ-OFF the distribution takes a large excursion, as if a large drift component appears, often stretching over 10 μV . We first considered that DMM must be disturbed by external noises and accordingly tried to reduce the effects to as low a level as possible. But the state of affairs was not improved at all. Another troublesome problem which weighed heavily on our mind was of the choice of number of power line cycles (NPLC). Certainly, the larger the number chosen, the higher the precision appeared to be, but the situation was not so simple. We therefore found that high precision measurements of μV order values were difficult, for a way of taking the statistical average of a lot of measured values obtained in a time sequential form was not so effective, and much improvement in practice was not expected even with parallel

measurements using several units of DMMs. Almost the same was also experienced in the HP3458A. This type is of the same as 3457A in its fundamental measurement scheme but we suspect that its additional artificial signal processing procedures make its situation more complicated.

We directed our attention to the characteristics of DMM itself: testing whether or not such nonrepeatability is intrinsic in every DMM. Under conditions such that the variations of all external influences are negligibly small, which we call the reference condition, we investigated in detail relatively long, time sequential observations obtained of a zero voltage state (the input terminals are short circuited, which we call zero voltage error) on the lowest or the next lowest measurement range, using four 3457A and two 3458A DMMs, with both AZ-ON and OFF. In the process of examining the nonrepeatability, we gradually became aware of two facts: (i) behaviors of the zero voltage error were in a statistical sense almost the same as what we had experienced when we had tried to evaluate the performances of the sensor and of the core noise measurement system developed by us and (ii) the statistically averaged value, in spite of increasing the measured values, fell into stagnation without approaching zero, no matter what NPLC was chosen. In relation to this, we tried to examine whether or not the same thing occurs on other measurement ranges. The result was common to every measurement range. The zero voltage error at AZ-ON fluctuates at least over the last one or two digits, while at AZ-OFF over the last two or three irrespective of NPLC chosen even for $\text{NPLC} \geq 1$.

The purpose of this article is to investigate the properties of the zero voltage state obtained in time sequential form, as being intrinsic to these DMMs. We first tried to directly treat the sequential observations as obtained under conditions in which they are a well-known stochastic time series, but sev-

eral important doubts arose on such a treatment. Such a situation compelled us to take a nonparametric treatment. The principal reason was: (i) A prolonged time interval is required for a data acquisition. Therefore, whether or not each element included in one measurement set is affected by all external influences has to be checked, that is, to test the randomness. (ii) How to represent a model of these DMMs for dealing with the zero voltage generation is not known, and (iii) whether the way of zero error correction for AZ-ON is appropriate or not, were also questions of interest.

Conventional measurement theories for precision, error, and accuracy reflect experiences from older electronics, but the development of high precision type DMMs seems to suggest that, in addition to this experience, some properties inherent in each DMM still have to be characterized before utilizing it fully. This article represents an extension to the subject matter. Our result leads to a proposal of a measurement procedure that extends the abilities of these DMMs, even when the experimental environment is not satisfactory (i.e., a sufficiently shielded area is not obtainable). Our experimental space is not shielded at all, and external noise in general is quite large. We can choose the time when the influence quantities are in a quiet state. Our conclusions suggest that some DMM properties are not special cases but rather occur in every place in which a DMM like this is employed. We are convinced that the proposed procedure works effectively under uncontrolled conditions. Here, our discussion is relevant to the case in 3457As on the measurement range of 0.3 V. NPLC=1 is chosen, as this level results in a large nonrepeatable difference. An observation number for a continuous sequence of $N=10\,000$ is chosen as the number required for characterizing AZ-OFF.

On assuming the application of the existing theory to the observation data obtained as a time sequential form, two things are tacitly required: each reading is (i) a sample from the same population and (ii) is independent of other readings. The error theory essentially pertains to multiple measurements,¹⁰ but often DMM are used for only a single measurement. We investigate whether or not the datum at each AZ-ON and OFF remains statistically under control, i.e., stable, and whether or not the measurement results cluster around the same central value and have the same variance (Robinovich's book¹⁰ p. 100, 4.6.). For this purpose, a conceptual model for the generation of zero voltage error in DMM is presented. It consists of two parts: an input voltage amplitude adjustment circuit (IVAAC) followed by a signal processing circuit (SPC). Each part individually generates error voltage. At AZ-ON, the zero error components in the SPC are eliminated.

Our results of AZ-ON tests are summarized as follows: (A) When viewing the data $N=10\,000$ on the whole, (i) the test of randomness based on the theory of runs suggests that each realization is produced from the same population; (ii) then the mean value as the best estimate of the data never converges to zero but works as a sort of systematic error. The cause is due to the existence of a preamplifier included in IVAAC; (iii) the voltage nearly equal to the median seems to be treated unusually and the evidence is seen near the highest point of the frequency distribution, and (iv) the greater part

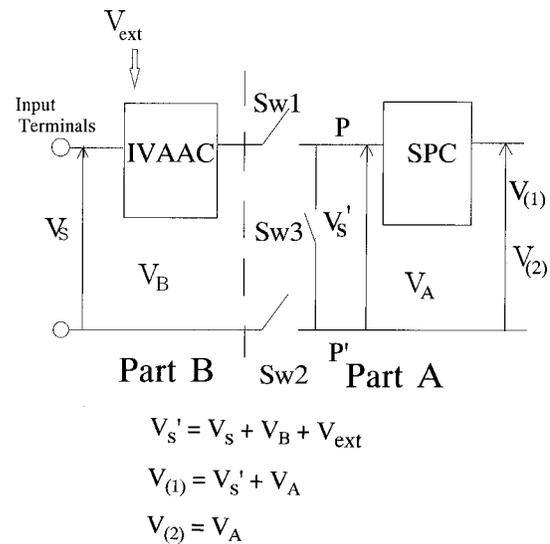


FIG. 1. Conceptual diagram representing the operation of DMM.

of the random fluctuation components in zero voltage error occurs at the output side of the preamplifier. (B) When examined by dividing the sequential data into several small partitioned parts, (i) ‘‘homogeneity’’ (statistical stability) is not sustained; (ii) therefore treating each part as a measurement set in a repetition measurement is not appropriate, and (iii) even if it is in a quiet area, any improvement in accuracy is not expected. Our viewpoint on the best use of sequential data at AZ-ON is that it will be suitable for evaluation of effects of the preamplifier on zero voltage error and the degree of mixture of any external influence quantities into a DMM. (These details are not presented here.) The external influence is not so large under the reference conditions. Therefore, it is treated here as part of the DMM.

With respect to AZ-OFF as pointed out above, the zero voltage error has such a form that small amplitude components fluctuate around large and slow excursions. Therefore, whether or not they are inherently generated from within the DMM itself is investigated. Simultaneously, what relationships exist between them at each AZ-ON and OFF is clarified. At AZ-OFF the voltage in the SPC just before starting a measurement is measured only once and subtracted from each subsequent measured value. As four DMMs (or six DMMs if including two 3458A) always reveal similar behaviors independent of one another under the reference conditions, we conclude that the zero voltage error at AZ-OFF is generated from within the DMM itself and is inherent in all these DMMs. The histogram shows an unusual frequency of two points of the voltage. The time sequence is similar to a discrete Gaussian process and is analyzed as a difference process. The sequence of the variance difference term has high homogeneity even in any partitioned parts, and this leads to a statistical and time series property suitable for application of error theory.

II. A MODEL FOR ZERO ERROR

A. Model for zero voltage error

A model shown in Fig. 1 is assumed as representing the

zero voltage error generation when a dc voltage is measured. A DMM consists of two parts: an IVAAC including a pre-amplifier of gain 100, and a SPC including sample-hold, double integral, and analog-to-digital transform circuits. Between them are internal switches SW1, 2, and 3. Since we do not know the details, this model does not quite exactly simulate the DMM but is rather a conceptual diagram. The signal to be measured v_s is first directed to IVAAC from the input terminal, to adjust v'_s for the range chosen, its full scale within the range of $\pm 9.999 \cdot V$, and finally to the SPC. The adjusted voltage v'_s appearing at terminals $P-P'$, when the input signal v_s is equal to that full scale value ($v_s=0.3$ or 0.03 V), becomes equal to $\pm 9.999 \cdot V$. These two parts, part A (SPC) and part B (IVAAC), generate respective internal zero voltage errors v_A and v_B . The resultant $v_A + v_B$ appears as the digitized output.

A state AZ-ON consisting of two modes per measurement is explained as follows: (i) v_s is first measured as $v_{(1)}$, where switches SW1 and SW2 are closed and SW3 is open [state (1)]; (ii) subsequently for zero adjustment SW1 and SW2 are opened, and SW3 is closed [state (2)], the zero correction $v_{(2)}$ is produced. But $v_{(1)}$ and $v_{(2)}$ never meet our eyes. Instead, a corrected value v_{on} comes into sight

$$v_{on} = v_{(1)} - v_{(2)} \tag{1}$$

as the quantity to be measured. There exists an inevitable time difference between measurements of $v_{(1)}$ and $v_{(2)}$. Ignoring time effects, the total is assumed as

$$v_{(1)} = v_s + v_A + v_B + v_{ext}, \tag{2}$$

$$v_{(2)} = v_A, \tag{3}$$

where v_{ext} represents an external voltage from the ambient conditions. Thus the measured output v_{on} is given as

$$v_{on} = v_s + v_B + v_{ext}. \tag{4}$$

In contrast for the case of AZ-OFF, the reading v_{off} is assumed as

$$v_{off} = v_{(1)} - v_{ini}(\text{const.}), \tag{5}$$

where $v_{ini} = v_{A,ini}$, the zero voltage correction generated in part A, measured only one time before starting the measurement and treated as a constant value until reset. Here, the problem of discriminating v_B from v_{ext} is not treated, and v_{ext} in Eqs. (2) or (4) is accordingly included into v_B .

B. Measurement system for v_{on} and v_{off}

The data acquisition system for v_{on} and v_{off} is as follows. Four DMMs are triggered simultaneously and read by a computer. For NPLC=1, the time for a measurement is 1 s and about 3 h are required for the data number $N=10\,000$ per sequence.

III. STATISTICAL PROPERTIES OF v_{on} AND v_{off}

A. Time series characteristics of $\{v_{on}\}$

1. Time domain characteristics of $\{v_{on}\}_{10\,000}^1$

Figure 2(a) indicates time domain characteristics of $\{v_{on}\}$ ($=\{v_{on}\}_{10\,000}^1$, and $\{v_{on}\}_{10\,000}^1 = \{v_{on,1}, v_{on,2}, \dots,$

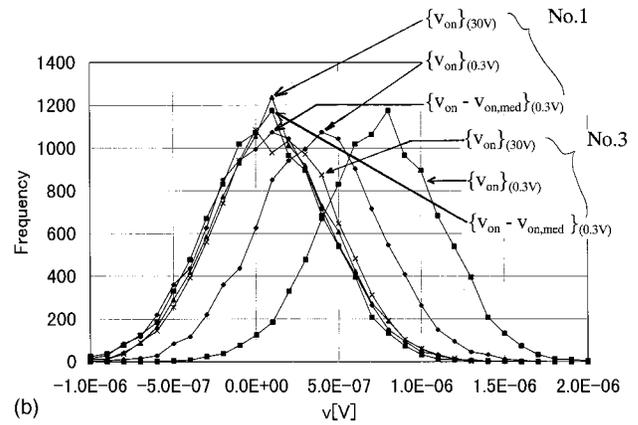
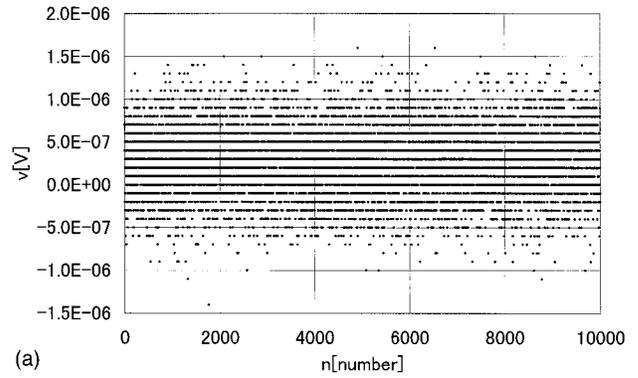


FIG. 2. Time domain characteristics and frequency distributions of $\{v_{on}\}$ and $\{v_{on}\}_{(30V)}$ at NPLC=1. (a) Time domain characteristics of $\{v_{on}\}$ for No. 1. (b) Frequency distributions.

$v_{on,10\,000}\}$) for measurement range of 0.3 V in DMM No. 1. For comparison, the case of range 30 V is similar but with a $100\times$ larger scale, and is directly related to the case of range 0.3 V. The measurable quantity in both ranges is first multiplied by one third through the same measurement channel in IVAAC. The multiplied quantity for range 30 V directly appears at the terminal $P-P'$ as v'_s , while the one for range 0.3 V passes through the preamplifier of gain 100 before appearing there. By comparing both ranges, effects of the preamplifier on the readings are evaluated. It is quite natural in a sense that $\{v_{on}\}$ and $\{v_{on}\}_{(30V)}$ ($\{v_{on}\}_{(30V)}$ indicates the case of range 30 V and “without subscript” for the case of 0.3 V), spread within ± 1.5 and $\pm 150 \mu\text{V}$, respectively. Their statistical parameters for No. 1 to No. 4 are listed in Table I, where v_{av} , v_{med} , $v_{s.d.}$, and v_{var} are respective mean, median, standard deviation (s.d.), and variance. The value of u given by

$$u = \frac{(v_{av} - \mu)}{\sqrt{v_{var}/n}} \tag{6}$$

is also calculated, where $\mu=0$ (the input terminal of DMM is short circuited) should be chosen, but here $\mu = v_{med}$ is taken. Table I teaches that the zero voltage correction at AZ-ON has the following properties: for s.d., the ratio of $v_{s.d.}$ for 0.3 V to the one for 30 V is almost equal to 1:100 in every DMM. In practice, the variance ratio obtained by multiplying v_{var} for 0.3 V by 10^4 remains within 1.104–0.978.

TABLE I. Statistical parameters in DMM Nos. 1–4. [Superscript number $(-p)$ means 10^{-p} .]

DMM	No. 1		No. 2		No. 3		No. 4	
Range	0.3 V	30 V						
v_{av}	$2.74^{(-7)}$	$6.05^{(-7)}$	$5.84^{(-7)}$	$4.73^{(-6)}$	$6.59^{(-7)}$	$2.36^{(-6)}$	$2.52^{(-7)}$	$-6.20^{(-6)}$
v_{med}	$3.0^{(-7)}$	0.0	$6.0^{(-7)}$	0.0	$7.0^{(-7)}$	0.0	$3.0^{(-7)}$	$-1.0^{(-5)}$
$v_{s.d.}$	$3.76^{(-7)}$	$3.64^{(-5)}$	$4.37^{(-7)}$	$4.16^{(-5)}$	$3.64^{(-7)}$	$3.65^{(-5)}$	$3.67^{(-7)}$	$3.72^{(-5)}$
v_{Var}	$1.41^{(-13)}$	$1.32^{(-9)}$	$1.91^{(-13)}$	$1.73^{(-9)}$	$1.32^{(-13)}$	$1.33^{(-9)}$	$1.35^{(-13)}$	$1.38^{(-9)}$
u	-6.92	1.66	-3.66	11.37	-11.26	6.47	-11.26	10.22

Judging from an F -test on the admissibility of differences between their variances, $\{v_{on}\}$ may be different from $\{v_{on}\}_{(30V)}$. However, when we perform this F -test to the properly partitioned subgroup pair obtained from these two, for example, between $\{v_{on}\}_{k+120}^k$ and $\{v_{on}\}_{k+120,(30V)}^k$ or $\{v_{on}\}_{k+240}^k$ and $\{v_{on}\}_{k+240,(30V)}^k$ for $k=1-9800$ (where $\{v_{on}\}_{k+120}^k$ means $\{v_k, v_{k+1}, \dots, v_{k+120}\}$), the result suggests that the null hypothesis that they belong to the same population is acceptable for almost all k (The choice of element number 120 or 240 is the maximum number of degrees of freedom in the F distribution tables at hand and this too small number does not give any definite conclusion as will be known later from the properties of $\{v_{on}\}$ or $\{v_{on}\}_{(30V)}$). It is therefore quite reasonable to suppose that $\{v_{on}\}$ and $\{v_{on}\}_{(30V)}$ are each subsets of the same population $\{v_B\}$. As mentioned above, only one point in which $\{v_{on}\}$ differs from $\{v_{on}\}_{(30V)}$ is the existence of the preamplifier in IVAAC. Another point for us to bear in mind is that $\{v_{on}\}$ is obtained at one time, while $\{v_{on}\}_{(30V)}$ is at another. Nevertheless, the F -test leads to such a conclusion. This makes us possible to evaluate the effects of the preamplifier on v_{on} qualitatively. When v_{on} is divided into three components $v_{on} = v_{on,1} + v_{on,2} + v_{on,3}$, where $v_{on,1}$ being the components generating in the input side of the preamplifier, $v_{on,2}$ within the preamplifier and $v_{on,3}$ in its output side, the greater part of v_{on} is composed of $v_{on,3}$, and $v_{on,2}$ dominates the components of which the mean value does not become equal to zero. $v_{on,1}$ can be treated as zero. Frequency distributions for $\{v_{on}\}$ and $\{v_{on}\}_{(30V)}$ on both Nos. 1 and 3 shown in Fig. 2(b), where measurement value as abscissa and the number of times each value as ordinate and $v_{on,(30V)}$ is multiplied by one-hundredth, back up this. For $\{v_{on}\}$ the curve of $\{v_{on} - v_{on,med}\}$ shifted by the value equal to each v_{med} is indicated together and is certainly in good agreement with that of the multiplied $\{v_{on}\}_{(30V)}$. This form is common to all these DMMs and the range over which the population $\{v_B\}_{(30V)}$ occupies remains unchanged to all of them. In this meaning $\{v_{on}\}$ represents intrinsic zero error of these DMMs. Another point to be noticed, also common to all these histograms, is an irregularity in the region of the peak frequency (at $0.1 \mu V$). The shape of the curve is never natural. We feel that the artificial signal processing must be adding to the measured values (see the histograms for HP3458As shown in Fig. 3(b), for they verify this more clearly). If we can understand it, ways of dealing with $\{v_{on}\}$ will become a little different. In any case, this leads us to a nonparametric treatment for investigating whether or not the probabilistic and statistical treatment of these residual voltages is reasonable, or how to apply the error theory is (similar irregularity is observed in

$\{v_{off}\}$). For reference, a set of data for two 3458As corresponding to Fig. 2 is indicated in Fig. 3, where Fig. 3(a) represents the time domain characteristics for the measurement range of 1 V on No. 1 and Fig. 3(b) the frequency distributions for the measurement ranges of 1 and 100 V on Nos. 1 and 2, respectively. The same procedure as in Fig. 2(b) is taken for the zero voltage error in the range 100 V. Data number $N=10\,000$ and $NPLC=1$ are similarly chosen. Whether or not the same relationship as in 3457A holds is checked, but it is not so. This is the reason why our attention is first directed to 3457A, which is much simpler and less complicated than a 3458A.

Summarizing the results: (i) the realization interval of $\{v_{on}\}$ is invariant and has not so large a difference among DMMs, so long as the path through which the signal passes is fixed. The limits are determined by the zero voltage components generated at the output side of the preamplifier. $\{v_{on}\}$ represents intrinsic zero offset in these DMMs. The

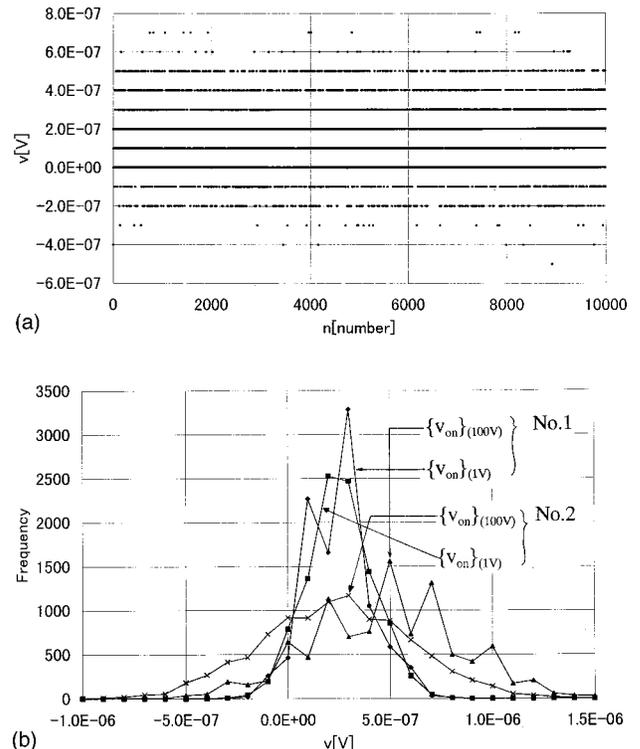


FIG. 3. Time domain characteristics and frequency distributions of $\{v_{on}\}_{(1V)}$ and $\{v_{on}\}_{(100V)}$ at $NPLC=1$ for DMM HP3458As. (a) Time domain characteristics of $\{v_{on}\}_{(1V)}$ for 3458A No. 1. (b) Frequency distributions.

TABLE II. Results on run occurrence in $\{v_{on}\}_{10\,000}^1$ in 0.3 and 30 V ranges.

		$m_{n,1}$	$m_{n,2}$	$m_{n,r}$	Mean ($\mu_{n,r}$)	Ratio ($m_{n,r}/\mu_{n,r}$)	Variance ($\sigma_{n,r}^2$)	$\mu_{n,r}$
No. 1	$\{V_{on}\}_{(30\text{ V})}$	4383	4379	4372	4382	0.9977	2190.3	-0.234
	$\{V_{on}\}$	4226	4700	4364	4451.41	0.9804	2218.7	-1.856
	$\{\epsilon_n\}$	4550	4461	5970	4506.06	1.3249	2252.06	30.848
	$\{\epsilon_{2n}\}$	2243	2263	2280	2253.96	1.0116	1126.21	0.7761
	$\{V_{on,diff}\}$	4595	4600	6252	4598.5	1.3596	2298.5	34.489
No. 3	$\{V_{on}\}_{(30\text{ V})}$	4737	4287	4380	4500.4	0.9732	2243.9	-2.373
	$\{V_{on}\}$	3961	4865	4355	4367.7	0.9971	2160.2	-0.273
	$\{\epsilon_n\}$	4478	4453	5974	4466.47	1.3375	2232.46	31.906
	$\{\epsilon_{2n}\}$	2236	2216	2165	2226.96	0.9722	1112.71	-1.857
	$\{V_{on,diff}\}$	4677	4577	6432	4627.46	1.39	2312.71	37.524

preamplifier produces a nonzero offset v_{med} in 0.3 V working as a sort of systematic error. In practice, v_{med} in 30 V is zero. (ii) The term u in Eq. (6) suggests that in most cases, v_{av} cannot be the best estimate of $\{v_{on}\}$ although it is for $\{v_{on}\}_{(30\text{ V})} \cdot v_{av}$ or v_{med} for 0.3 V cannot be neglected as zero, whereas v_{av} for 30 V which is of the order 10^{-6-7} can be regarded zero, for it is beyond the resolution of that range. (iii) The 0.3 V range is not suitable to such small voltage measurement in a sense, for the output is unable to keep off the mixture of zero voltage offset into the signal to be measured, and the same thing occurs in the 0.03 V range.

2. Statistical and probabilistic properties of $\{v_{on}\}$

As pointed out above, both the histogram of $\{v_{on}\}$ and $\{v_{on}\}_{(30\text{ V})}$ seem to be a non Gaussian. In addition to this, one point to be noticed is that it is quite doubtful, though will be concretely discussed later from a little different angle, whether the outcome $v_{on,k}$ generated at each instant of time $k(k=1-10\,000)$ has a specified probability distribution, that is, is a sample from the same population. In other words, it is unclear whether or not each element in the ordered sequence occurs purely randomly, or constitutes a stochastic time series with a form described in any books on the conventional probability theory. The ‘‘randomness’’ means to test such a hypothesis that n observations x_1, x_2, \dots, x_n , when they are obtained in this order, satisfy the above condition. In such a case, the theory of runs^{11,12} is applied in order to check it. The procedure is; Comparing each $v_{on,k}$ with $v_{on,med}$ ($v_{on,med} = \text{median of } \{v_{on}\}_{10\,000}$), replace $v_{on,k}$ by α if $v_{on,k} > v_{on,med}$ and by β if $v_{on,k} < v_{on,med}$, or otherwise 0. Consequently, $\{v_{on}\}$ is replaced by the sequence of α and β , or 0. (Here, the occurrence of 0 is neglected for simplicity.) Then each maximal subsequence of elements of like kind α or β is called a run. The randomness of $\{v_{on}\}$ and $\{v_{on}\}_{(30\text{ V})}$ is evaluated how these runs of α and β occur. When $m_{n,1}$, $m_{n,2}$, and $m_{n,r}$ are the numbers of symbols α , β , and runs, respectively, $m_{n,1}$ and $m_{n,2}$ give the mean value $\mu_{n,r}$ for occurrence number $m_{n,r}$ of runs and the corresponding variance $\sigma_{n,r}^2$ as follows:

$$\mu_{n,r} = E(m_{n,r}) = \frac{2m_{n,1}m_{n,2}}{m_{n,1} + m_{n,2}} + 1, \tag{7}$$

$$\sigma_{n,r}^{(2)} = \frac{2m_{n,1}m_{n,2}(2m_{n,1}m_{n,2} - m_{n,1} - m_{n,2})}{(m_{n,1} + m_{n,2})^2(m_{n,1} + m_{n,2} - 1)}, \tag{8}$$

and for $m_{n,1} + m_{n,2}$ larger than 20, the occurrence probability of $m_{n,r}$ is approximated by a normal distribution. That is, the variable $u_{n,r}$ derived by

$$u_{n,r} = \frac{m_{n,r} - \mu_{n,r}}{\sigma_{n,r}} \tag{9}$$

approximately obeys the standard normal distribution $N(0,1)$. Then, when $m_{n,1}$ and $m_{n,2}$ are sufficiently large and $m_{n,1} = m_{n,2}$ holds, $\sigma_{n,r}$ takes its maximum value and $\sigma_{n,r}^2/(m_{n,1} + m_{n,2})$ is about to arrive at 0.25, while $\mu_{n,r}/(m_{n,1} + m_{n,2})$ approaches 0.5. Although it is most important how the runs occur (the detail is not presented here), it can at least be said that, when the ratio $m_{n,r}/\mu_{n,r}$ approaches 1 and $\sigma_{n,r}/(m_{n,1} + m_{n,2})$ does 0.25, the randomness is high. A result examined like this for $\{v_{on}\}$ and $\{v_{on}\}_{(30\text{ V})}$ is indicated together with others in Table II. Table II poses an interesting viewpoint to us. $m_{n,r}$ is close to the corresponding mean value: the ratio of $m_{n,r}$ of $\mu_{n,r}$ ($= m_{n,r}/\mu_{n,r}$) being

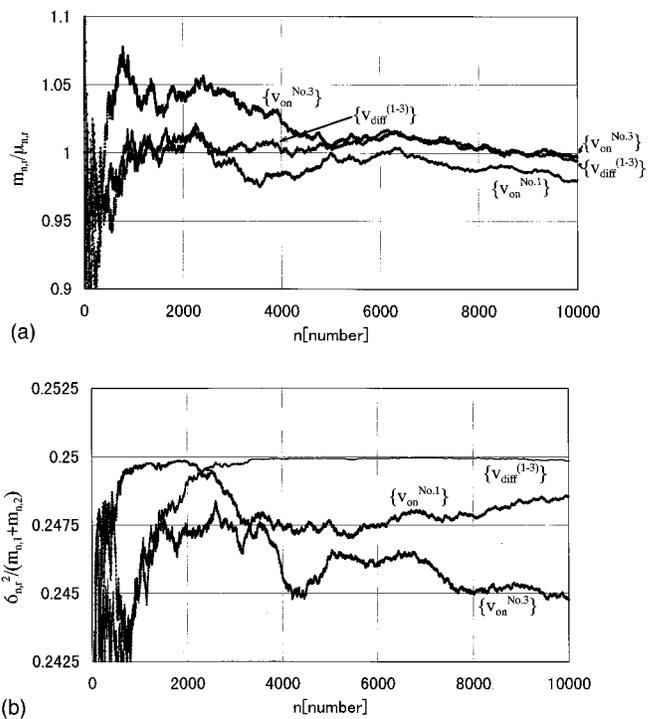
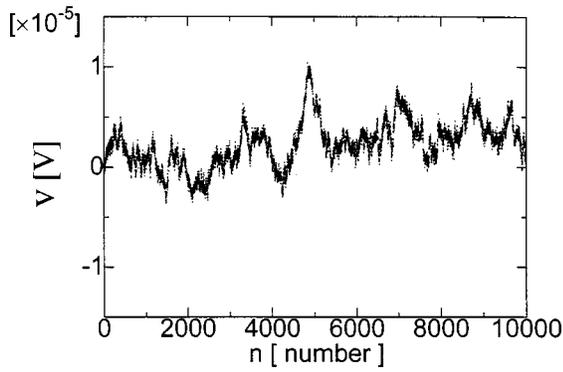
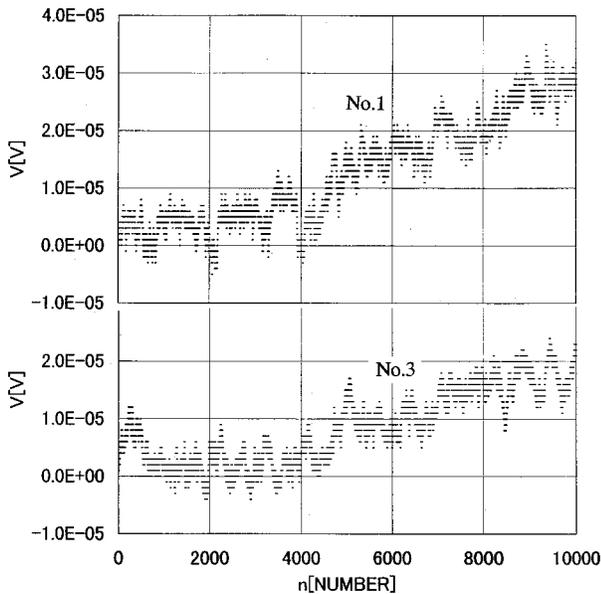


FIG. 4. An evaluation of v_{ext} by means of run. (a) Occurrence number vs n . (b) s.d. vs n .

FIG. 5. Time domain characteristics of $\{v_{\text{off}}\}$ at NPLC=1 for No. 1.

almost equal to 1 and $u_{n,r}$ in Eq. (9) remains within -2.37 to -0.234 . The randomness of both ordered sequences $\{v_{\text{on}}\}$ and $\{v_{\text{on}}\}_{(30\text{ V})}$ is high. They are separately obtained in each different time, but can be regarded as two groups of random quantities belonging to the same population $\{v_B\} \cong \{v_{\text{on},3}\}$ as shown in Fig. 1. The preamplifier does not play a role in offsetting $\{v_{\text{on},3}\}$. If the component pointed out as a sort of systematic error, $v_{\text{on},2}$, behaves in a randomly fluctuating manner, the aspects of run occurrence in $\{v_{\text{on}}\}$ drastically differ from that in $\{v_{\text{on}}\}_{(30\text{ V})}$. $v_{\text{on},2}$ must be nearly constant or slowly varying. We cannot help but recognize that the hypothesis of randomness is acceptable irrespective of the existence of non-Gaussian components in their frequency distributions.

The fact that both $\{v_{\text{on}}\}$ and $\{v_{\text{on}}\}_{(30\text{ V})}$ are of the same degree of randomness and dominated by $\{v_{\text{on},3}\}$ indirectly verifies that effects of v_{ext} on these DMMs are negligibly small under the reference conditions, although $\{v_{\text{ext}}\}$ dominates $\{v_{\text{on},1}\}$. In practice, Fig. 4 suggests this, where run occurrence characteristics of $\{v_{\text{on}}^{\text{No.1}}\}$, $\{v_{\text{on}}^{\text{No.3}}\}$, and $\{v_{\text{diff}}^{(1-3)}\}$ are indicated, which respectively represent $\{v_{\text{on}}\}$ from re-

FIG. 6. Time domain characteristic of $\{v_{\text{off}}\}_{(3\text{ V})}$ which seems to respond to faint room temperature variation.TABLE III. Statistical parameters of $\{v_{\text{off}}\}$ on 0.3 V in Nos. 1–4.

DMM	No. 1	No. 2	No. 3	No. 4
v_{av}	2.32×10^{-6}	3.09×10^{-6}	8.28×10^{-7}	5.49×10^{-7}
v_{med}	2.4×10^{-6}	3.0×10^{-6}	9.0×10^{-7}	6.0×10^{-7}
$v_{\text{s.d.}}$	2.31×10^{-6}	1.87×10^{-6}	1.71×10^{-6}	1.06×10^{-6}
v_{Var}	5.33×10^{-12}	3.48×10^{-12}	2.92×10^{-12}	1.12×10^{-12}

spective DMM No. 1 and No. 3 and difference voltage $v_{\text{diff}}^{(1-3)}$ between $v_{\text{on}}^{\text{No.1}}$ and $v_{\text{on}}^{\text{No.3}}$ given by

$$v_{\text{diff},k}^{(1-3)} = v_{\text{on},k}^{\text{No.1}} - v_{\text{on},k}^{\text{No.3}} = v_{B,k}^{\text{No.1}} - v_{B,k}^{\text{No.3}}. \quad (10)$$

$\{v_{\text{diff}}^{(1-3)}\}$ does not include the term of v_{ext} in Eq. (4), for v_{ext} is treated as common to all DMMs. The ratio $m_{n,r}/\mu_{n,r}$ and the variance ratio $\sigma_{n,r}^2/(m_{n1} + m_{n2})$ are depicted as a function of the sampling time n in Figs. 4(a) and 4(b), respectively. In every case, the former is going to approach 1 and the latter to 0.25 for $n > 500 - 1000$. These three suggest they have almost the same degree of randomness and each element belonging to both $\{v_{\text{on}}^{\text{No.1}}\}$ and $\{v_{\text{on}}^{\text{No.3}}\}$ is independently produced with one another without being affected by external influence quantities. However, more strictly speaking, a slight difference arises in $v_{\text{diff}}^{(1-3)}$, where the degree of randomness becomes stronger than the other two, for $\sigma_{n,r}^2/(m_{n1} + m_{n2})$ is more close to 0.25 and the equality $m_{n1} = m_{n2}$ holds for almost all $n \geq 3000$ and $\sigma_{n,r}$ takes its maximum value. Such a tendency becomes more remarkable for larger n .

B. Characteristics of $\{v_{\text{off}}\}_{10\text{ 000}}$

1. Time domain characteristics of $\{v_{\text{off}}\}_{10\text{ 000}}$

Time series characteristics of $\{v_{\text{off}}\}$ is indicated in Fig. 5 for No. 1. The variability for $\{v_{\text{off}}\}_{(30\text{ V})}$ spreads over 100 times that for $\{v_{\text{off}}\}$; from $-5 - 10 \mu\text{V}$ to $-0.5 - 1 \text{ mV}$. Two frequency components are observed, a relatively slow with a large amplitude and a fast fluctuating one. From Eqs. (4) and (5), v_{off} includes v_A , while v_{on} does not. Comparison of Fig. 5 with Fig. 2(a) suggests that $\{v_A\}$ seems to be the former slow component, common to every range for AZ-OFF in all DMMs, while $\{v_B\}$ is the latter. The model shown in Fig. 1 assumes this. Figures 2, 5, and 7 indicate that $\{v_{\text{on}}\}$ is noise in the lowest two digits, while $\{v_{\text{off}}\}$ the lowest three digits. The tendency like this does not depend on measurement range chosen.

The reason for choosing the data number $N = 10\text{ 000}$ is to check whether or not the large excursion component is simply a drift caused by faint room temperature or other variations. If so, any clear correlation between $\{v_{\text{off}}^{\text{No.1}}\}$ and $\{v_{\text{off}}^{\text{No.3}}\}$ should be observed, but, in practice, not so. Figure 6 is such an example for $\{v_{\text{off}}\}_{(3\text{ V})}$ at NPLC=10, where a generally increasing component is apparent in both $\{v_{\text{off}}^{\text{No.1}}\}$ and $\{v_{\text{off}}^{\text{No.3}}\}$ with the deviation magnitude of about $20 \mu\text{V}$.

Statistical parameters on $\{v_{\text{off}}\}$ are listed in Table III and frequency distributions corresponding to Fig. 5 are indicated in Fig. 7, where common to all DMMs the frequency at $v_{\text{off}} = 0.0 \mu\text{V}$ is unusually high, while that at $v_{\text{off}} = 1.3 \mu\text{V}$ is reduced to zero. The cause is within 3457A itself.

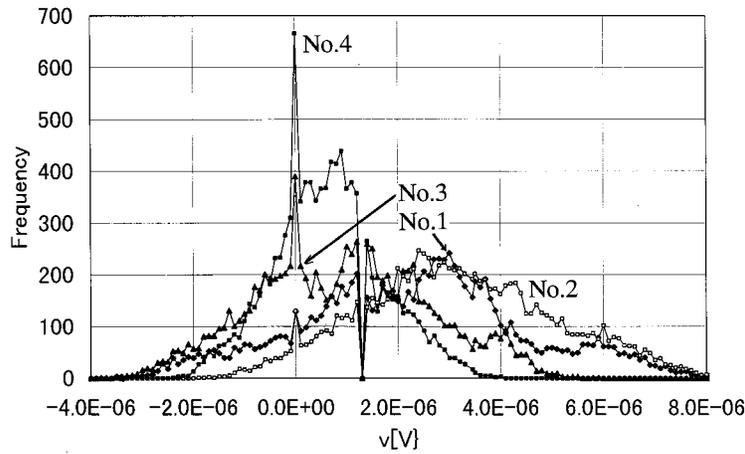


FIG. 7. Frequency distributions of $\{v_{off}\}$.

2. Characteristic representation of $\{v_{off}\}^{10\ 000}$ in the variate difference plane $(v_{off,n}, v_{off,n+1})$

$\{v_{off}\}$ seems to closely resemble a Gaussian process in appearance. If it is so, then any difference term $v_{off,n+k} - v_{off,n}$ for any positive integer k obeys a normal distribution and adjacent difference terms $v_{off,2} - v_{off,1}, v_{off,3} - v_{off,2}, \dots, v_{off,n+1} - v_{off,n}$ are independent with one another.¹³ Taking this into consideration, we represent $\{v_{off}\}$ in a plane $(v_{off,n}, v_{off,n+1}), v_{off,n}$ as abscissa and $v_{off,n+1}$ as ordinate, in Fig. 8. v_{off} scatters around the straight line $y = x (v_{off,n+1} = v_{off,n})$ passing through the origin and this signifies that $v_{off,n+1}$ can be written as

$$v_{off,n+1} = v_{off,n} + \epsilon_{n+1}, \tag{11}$$

where ϵ_{n+1} is also plotted together with $(v_{off,n}, v_{off,n+1})$, which represents the adjacent difference term. If $\{\epsilon_{n+1}\}$ satisfies the above two conditions (every ϵ_{n+1} belongs to a population of a normal distribution and is probabilistically independent with each other), the situation is quite simplified. Therefore, its statistical and probabilistic properties including randomness are investigated in detail instead of directly studying $\{v_{off}\}$.

On the other hand, the large excursion component in $\{v_{off}\}$ is in either an increasing or decreasing state. As shown

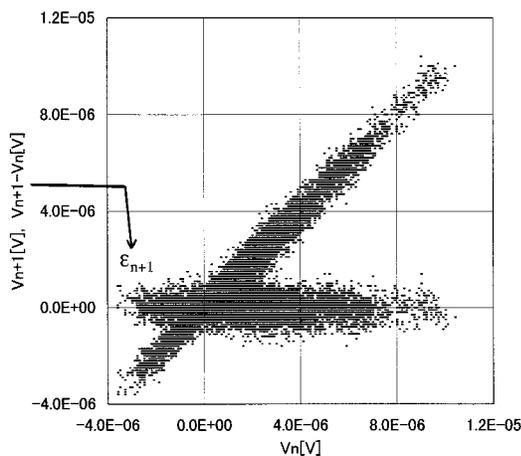


FIG. 8. Characteristics of $\{v_{off}\}$ and $\{\epsilon\}$ in the plane $(v_{off,n+1}, v_{off,n})$ for No. 1.

in Fig. 9, picking up a part where $v_{off,n}$ is in an increasing state with the slope α_{n1} in a region from $n = n_1$ to $n = n_2$, then v_n (subscript ‘‘off’’ is deleted.) at time n in $n_1 \leq n \leq n_2$ is represented as

$$v_n = v_{A,n} + v_{B,n} \tag{12}$$

$$= (n - n_1)\alpha_{n1} + v_{B,n} + v_{A,n1}, \tag{12a}$$

that is,

$$v_{A,n} = (n - n_1)\alpha_{n1} + v_{A,n1}. \tag{13}$$

Using Eq. (12), v_{n+1} is related to v_n as

$$v_{n+1} = (n + 1 - n_1)\alpha_{n1} + v_{B,n+1} + v_{A,n1} \tag{14}$$

$$= v_n + (v_{B,n+1} - v_{B,n}) + \alpha_{n1}, \tag{15}$$

where

$$\epsilon_{n+1} = (v_{B,n+1} - v_{B,n}) + \alpha_{n1}. \tag{16}$$

From Eq. (11) v_{n+1} is represented as a cumulative of $\{\epsilon\}_{n+1}^2$ or of $\{\alpha\}$ with equal weights

$$v_{n+1} = v_1 + \sum_{k=2}^{n+1} \epsilon_k, \tag{17}$$

$$v_{n+1} = (v_{B,n+1} - v_{B,1}) + \sum \alpha_{nk}. \tag{17a}$$

Equations (17) and (17a) suggest that there are two ways of characterizing $\{v_{off}\}$: (i) as α_{nk} has nearly a constant value

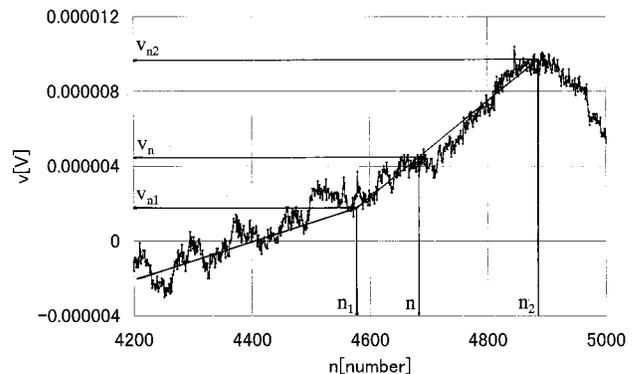


FIG. 9. Determination of Eq. (11).

TABLE IV. Statistical quantities of $\{\epsilon_n\}_{10\,000}^2$ and $\{v_{on}\}_{10\,000}^1$. (The case of $\{v_{on,dif}\}_{10\,000}^2$ is included.)

		Mean	s.d.	Var	$\sigma_\epsilon/\sigma_{v,on}$	$\sigma_{on,dif}/\sigma_{v,on}$	$\sigma_\epsilon/\sigma_{on,dif}$
			($\times 10^{-7}$)	($\times 10^{-13}$)			
No. 1	$\{\epsilon_n\}$	1.7×10^{-10}	4.11	1.68	1.093	1.404	0.778
	$\{\epsilon_{2n}\}$	7.80×10^{-9}	4.12	1.70			
	$\{v_{on,dif}\}$	-9.00×10^{-11}	5.28	2.79			
	$\{v_{on} - v_{on,med}\}$	-0.26×10^{-7}	3.76	1.41			
No. 2	$\{\epsilon_n\}$	1.9×10^{-10}	4.95	2.45	1.133	1.410	0.804
	$\{\epsilon_{2n}\}$	-5.88×10^{-7}	4.92	2.42			
	$\{v_{on,dif}\}$	-8.00×10^{-11}	6.16	3.79			
	$\{v_{on} - v_{on,med}\}$	-0.16×10^{-7}	4.37	1.91			
No. 3	$\{\epsilon_n\}$	1.9×10^{-10}	3.90	1.52	1.071	1.401	0.765
	$\{\epsilon_{2n}\}$	1.80×10^{-9}	3.88	1.51			
	$\{v_{on,dif}\}$	-5.00×10^{-11}	5.10	2.60			
	$\{v_{on} - v_{on,med}\}$	-0.41×10^{-7}	3.64	1.32			
No. 4	$\{\epsilon_n\}$	8.0×10^{-11}	3.75	1.41	1.02	1.398	0.731
	$\{\epsilon_{2n}\}$	1.24×10^{-9}	3.72	1.38			
	$\{v_{on,dif}\}$	2.00×10^{-11}	5.13	2.63			
	$\{v_{on} - v_{on,med}\}$	-0.48×10^{-7}	3.67	1.35			

in a certain interval of n and does not vary randomly, ϵ_{n+1} or $(v_{B,n+1} - v_{B,n})$ is dominant and (ii) $v_{off,n+1}$, independent of $v_{B,k}$ on the way to $n+1$, is dominated by either $v_{B,n+1}$ or $\sum \alpha_{nk}$ or both. Certainly, the term $(v_{B,n+1} - v_{B,n})$ has a large weight in ϵ_{n+1} . Roughly evaluating α_{nk} as determined in the above, it is within the range of $|\alpha_k| = 2-3 \times 10^{-8}$ (V/step) from Fig. 5. The purpose of AZ-ON is to eliminate the effects of v_A on the output reading. As pointed out before, for implementing the elimination, it should be presumed that there does not exist any difference caused by the very short time difference, i.e., $v_{A,n} - v'_{A,n} = 0$ can always be satisfied. α_{n1} is given by $\alpha_{n1} = (v_{A,n2} - v_{A,n1}) / (n_2 - n_1)$, or by an averaged value of $(v_{A,n+1} - v_{A,n})$, and is of the order $|\alpha_k| = 2-3 \times 10^{-8}$ (V/step). This magnitude is out of the resolution even for the range of 0.03 V. In this meaning, the role of AZ-ON is completely performed. $\{v_{on}\}$ certainly includes the term of $\{v_B\}$ only but is suppressed within $\pm 1 \mu\text{V}$ (see Fig. 2). From Eq. (16) the weight of α_κ in ϵ_{n+1} is not so large. Therefore, in ϵ_{n+1} given by

$$\begin{aligned} \epsilon_{n+1} &= v_{off,n+1} - v_{off,n} \\ &= \{v_{B,n+1} - v_{B,n}\} + \{v_{A,n+1} - v_{A,n}\}, \end{aligned} \quad (18)$$

the second term is forced into negligibly small quantities compared with the first at every sampling. Nevertheless, $\{v_{off}\}$ has large excursions. In addition to this, the range that $\{v_{B,n+1} - v_{B,n}\}$ takes is in general wider than that of $\{v_{B,n}\}$ and the same holds between $\{v_{A,n+1} - v_{A,n}\}$ and $\{v_{A,n}\}$. Noticing these relations, statistical and probabilistic properties of $\{v_{off}\}$, that is, $\{v_A\}$ and $\{v_B\}$, are subsequently clarified.

3. Frequency distribution of $\{\epsilon\}$

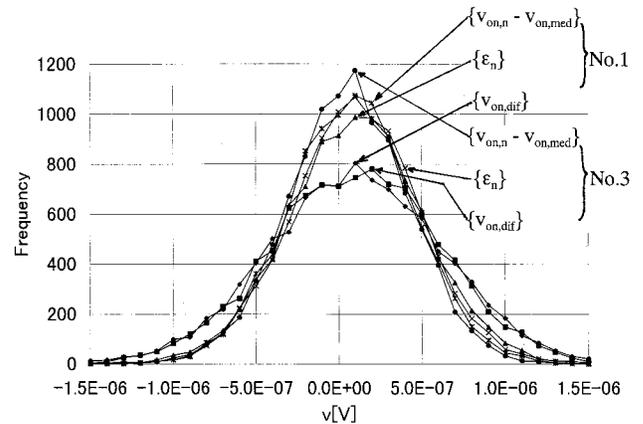
Statistical parameters of $\{\epsilon_n\}$ and $\{v_{on,dif}\} = \{v_{on,n+1} - v_{on,n}\}_{n=9999}^1 (= \{v_{B,n+1} - v_{B,n}\}_{n=9999}^1)$ are summarized in Table IV and their frequency distributions are indicated in Fig. 10, where for comparisons the case of $\{v_{on} - v_{on,med}\}_{10\,000}^1$ ($v_{on,med} = \text{median of } \{v_{on}\}_{10\,000}^1$) is also added. Looking at each shape around the peak frequency, a fairly clear correlation is observed between $\{\epsilon_n\}$ and $\{v_{on} - v_{on,med}\}$ in the same DMM. As a whole there does not exist

a large discrepancy between them. $\{v_{on,dif}\}$, however, is evidently different from the other two and spreads out more widely as known from Table IV. Equation (16) gives the following for mean values:

$$\epsilon_{n,av} = v_{on,dif,av} + \alpha_{nk,av}. \quad (19)$$

From Table IV their actual values are: $\epsilon_{n,av}$ = the order of 10^{-10} , $v_{on,dif,av}$ = of 10^{-11} . Those of $\{v_{on}\}$ (see Table I) and $\{v_{on} - v_{on,med}\}$ are of the order of 10^{-7} or 10^{-8} . $\epsilon_{n,av}$ is larger than $v_{on,dif,av}$ by one figure, and $\alpha_{nk,av}$ thus dominates it. Effects of the preamplifier on zero voltage can thus be completely eliminated in $\{v_{on,dif}\}$. By introducing a procedure of taking the difference between adjacent terms in $\{v_{B,n}\}$ it becomes possible. Therefore, the existence of $\{v_A\}$ or $\{\alpha_k\}$ as residual voltage is never small and the operation of AZ-ON is adequate as pointed out above.

Next, let's discuss the variances $v_{on,dif,Var}$, $v_{on,Var}$, and $\epsilon_{n,Var}$ of these sequences $\{v_{on,dif}\}$, $\{v_{on}\}$, and $\{\epsilon_n\}$. $v_{on,dif,Var}$ is related to $v_{on,Var}$ as

FIG. 10. Frequency distributions of $\{\epsilon_n\}$, $\{v_{on,dif}\}$, and $\{v_{on} - v_{on,med}\}$.

$$\begin{aligned}
 v_{\text{on,dif,Var}} &= \{v_{B,n+1} - v_{B,n}\} \text{Var} \\
 &= E\{((v_{B,n+1} - v_{\text{on,av}}) - (v_{B,n} - v_{\text{on,av}}))^2\} \\
 &= E\{(v_{B,n+1} - v_{\text{on,av}})^2\} + E\{(v_{B,n} - v_{\text{on,av}})^2\} \\
 &\quad - 2E\{(v_{B,n+1} - v_{\text{on,av}})(v_{B,n} - v_{\text{on,av}})\} \\
 &= 2v_{\text{on,Var}} - 2\text{Cov}\{v_{B,n+1}, v_{B,n}\} \quad (20)
 \end{aligned}$$

where $E(\{X_n\})$ indicates the arithmetic average of $\{X_n\}$ and $\text{Cov}\{v_{B,n+1}, v_{B,n}\}$ represents the covariance between the adjacent terms in $\{v_{\text{on}}\}$. Equation (16) derives $\epsilon_{n,\text{Var}}$ as

$$\begin{aligned}
 \epsilon_{n,\text{Var}} &= \{(v_{B,n+1} - v_{B,n}) + \alpha_{n1}\} \text{Var} \\
 &= E\{[(v_{B,n+1} - v_{\text{on,av}}) - (v_{B,n} - v_{\text{on,av}})] \\
 &\quad + (\alpha_{n1} - \alpha_{nk,\text{av}})\}^2 \\
 &= v_{\text{on,dif,Var}} + \alpha_{nk,\text{Var}} + 2\text{Cov}\{v_{\text{on,dif}}, \alpha_{nk}\} \quad (21)
 \end{aligned}$$

or Eq. (18) does it as

$$\begin{aligned}
 \epsilon_{n,\text{Var}} &= \{(v_{B,n+1} - v_{B,n}) + (v_{A,n+1} - v_{A,n})\} \text{Var} \\
 &= E\{[(v_{B,n+1} - v_{\text{on,av}}) - (v_{B,n} - v_{\text{on,av}})] \\
 &\quad + [(v_{A,n+1} - v_{A,\text{av}}) - (v_{A,n} - v_{A,\text{av}})]\}^2 \\
 &= v_{A,\text{dif,Var}} + v_{B,\text{dif,Var}} + 2\text{Cov}\{v_{A,\text{dif}}, v_{B,\text{dif}}\}, \quad (21a)
 \end{aligned}$$

where $\text{Cov}\{v_{\text{on,dif}}, \alpha_{nk}\}$ and $\text{Cov}\{v_{A,\text{dif}}, v_{B,\text{dif}}\}$ similarly are the covariance between $\{v_{\text{on,dif}}\}$ and $\{\alpha_{nk}\}$, and $\{v_{A,\text{dif}}\}$ and $\{v_{B,\text{dif}}\}$, respectively. From their actual values (see Tables I and IV) we can derive the following important results on these sequences. They are of the same order, but $\epsilon_{n,\text{Var}}$ is 105–130% of $v_{\text{on,Var}}$ and 50–60% of $v_{\text{on,dif,Var}} (= v_{B,\text{dif,Var}})$, and $v_{\text{on,dif,Var}}$ is twice $v_{\text{on,Var}}$. Equation (20) indicates that the following holds:

$$\text{Cov}\{v_{B,n+1}, v_{B,n}\} = 0 \quad (20a)$$

that is, the adjacent terms in $\{v_{\text{on}}\}$ are uncorrelated. On $\epsilon_{n,\text{Var}}$, Eqs. (21) and (21a) require: (i) the correlation between $\{v_{B,\text{dif}}\}$ and $\{v_{A,\text{dif}}\}$ is negative, that is, $\text{Cov}\{v_{\text{on,dif}}, \alpha_{nk}\} \leq 0$, so it will work to reduce $v_{\text{on,dif,Var}}$ and (ii) the magnitude is of the same order as in $v_{\text{on,dif,Var}}$, for $\epsilon_{n,\text{Var}} < v_{\text{on,dif,Var}}$ has to be satisfied (Here, we will only say that the correlation is strongly revealed when a small voltage of μV order or less is measured). Assuming that $\{v_{A,n}\}$ is uncorrelated with the adjacent terms, $\alpha_{nk,\text{Var}}$ is also given as twice that of $v_{A,\text{Var}}$. Table III gives $v_{\text{off,Var}}$ = the order of 10^{-12} . If $v_{\text{off,Var}} = v_{A,\text{Var}} + v_{B,\text{Var}}$ holds, $v_{A,\text{Var}}$ has to become dominant in $v_{\text{off,Var}}$, as the order of $v_{B,\text{Var}}$ being 10^{-13} . Consequently, the relationship $v_{\text{off,Var}} = v_{A,\text{Var}} + v_{B,\text{Var}} + \text{Cov}\{v_A, v_B\}$ has to hold as derived in Eq. (12) and $\text{Cov}\{v_A, v_B\}$ has to be positive and dominant in $\{v_{\text{off,Var}}\}$. However, so long as we adhere to the above evaluation of $\alpha_k (|\alpha_k| = 2-3 \times 10^{-8})$ based on Eqs. (12a)–(16), we cannot reasonably explain many things: for example, if α_k is of the order of $2-3 \times 10^{-8}$, the shape of histogram of $\{\epsilon\}$ is dominated by $\{v_{\text{on,dif}}\}$ and never close to $\{v_{\text{on}}\}$. Figure 10 suggests this. Although $\{v_A\}$ may be dependent on $\{v_B\}$, it is never small but rather has the same order of $\{v_B\}$. $\{v_A\}$ has ‘‘a power of randomness’’ which changes the histogram of

$\{v_{\text{on,dif}}\}$ into that close to $\{v_{\text{on}}\}$. Though AZ-ON aims at directly eliminating v_A , it is doubtful whether the procedure effectively works.

The noticeable point of $\epsilon_{n,\text{av}} \cong 0$ gives a hint to us for proposing a new way of achieving high precision measurement of μV order dc voltage. In practice, $\{\epsilon\}$ has several interesting properties, which will be investigated by dividing $\{\epsilon\}$ into two groups (the reason will be clarified in the proceeding sections); the total series $\{\epsilon_n\}_{10\,000}^2 = \{\epsilon_n : n = 2, 3, \dots, 10\,000\}$ and the set of even terms $\{\epsilon_{2n}\}_{10\,000}^2 = \{\epsilon_2, \epsilon_4, \epsilon_6, \dots, \epsilon_{10\,000}\}$ or that of odd ones $\{\epsilon_{2n+1}\}_{9999}^3 = \{\epsilon_3, \epsilon_5, \epsilon_7, \dots, \epsilon_{9999}\}$.

4. Randomness of $\{\epsilon_n\}_{10\,000}^2$ and $\{\epsilon_{2n}\}_{10\,000}^2$ or $\{\epsilon_{2n+1}\}_{9999}^3$

The result obtained on the randomness of $\{\epsilon_n\}_{10\,000}^2$ and $\{\epsilon_{2n}\}_{10\,000}^2$ is summarized in Table II together with that of $\{v_{\text{on,dif}}\}$. The ratio $m_{n,r} / \mu_{n,r}$ vs n makes an excursion around 1.33 for $\{\epsilon_n\}_{10\,000}^2$, but 1.0 for $\{\epsilon_{2n}\}_{10\,000}^2$ similar to those shown in Fig. 4. The same as in $\{\epsilon_{2n}\}_{10\,000}^2$ occurs in $\{\epsilon_{2n+1}\}_{9999}^3$. For $\{\epsilon_n\}u_r$ in Eq. (9) takes a value of nearly 30, while 1.0 to -1.86 for $\{\epsilon_{2n}\}$. In $\{\epsilon_n\}$ the meaning of u_r taking such a value is that it happens that such a case does seldom occur probabilistically. This is closely related to the fact that $m_{n,r} / \mu_{n,r}$ is close to 1.33. The meaning is that the number of runs is too many, that is, the term $\epsilon_n - \epsilon_{\text{med}}$ (median of $\{\epsilon_n\}$) almost takes a positive and a negative value alternatively with the increase of n . The run of length 1 occupies the greater part in $\{\epsilon_n\}$. That is, the randomness in $\{\epsilon_n\}$ is very low and the regularity is high. On the other hand, the randomness in $\{\epsilon_{2n}\}$ is high. Table II is very instructive on which of v_A or v_B dominates in $\{\epsilon_n\}$, the occurrence ratio in $\{v_{\text{on,dif}}\}$ being just equal to 1.35–1.38. The first term $(v_{B,n} - v_{B,n-1})$ in Eq. (18), as pointed out above, plays a deterministic role for characterizing the randomness of $\{\epsilon_n\}$, while the second one has a large influence on determination of the shape of histogram of $\{\epsilon_n\}$ but not so on the randomness. $\epsilon_{n,\text{Var}}$ is about 60% of $v_{\text{on,dif,Var}} (= v_{B,\text{dif,Var}})$ (see Table IV). $v_{A,\text{dif}}$ is apt to work to reduce $v_{B,\text{dif}}$. In other words, the probability that the former has the inverse sign of the latter but does not exceed its absolute value is high. However, in $v_{\text{off},n}$ the probability that $v_{A,n}$ has the same sign as that of $v_{B,n}$ is high. Nevertheless, $\{v_A\}$ has to include the components which produce large excursions in $\{v_{\text{off}}\}$.

Summarizing the results obtained in the above, for taking an average of observation data $\{v_{\text{on}}\}$ or $\{v_{\text{off}}\}$ for deriving its best estimate, one important requirement is its randomness. $\{v_{\text{on}}\}$ certainly satisfies this. But the so-called drift component has a large influence on zero voltage offset. Its effect is eliminated in $\{v_{\text{on,dif}}\}$, but randomness in $\{v_{\text{on,dif}}\}$ is low. Consequently, our attention is naturally directed to $\{\epsilon_n\}$ and $\{\epsilon_{2n}\}$. In particular, the randomness in $\{\epsilon_{2n}\}$ is fit for our purpose. Another requirement is the degree of correlation or orthogonality between the adjacent terms. If the correlation is high, the set is not suitable for taking its average.

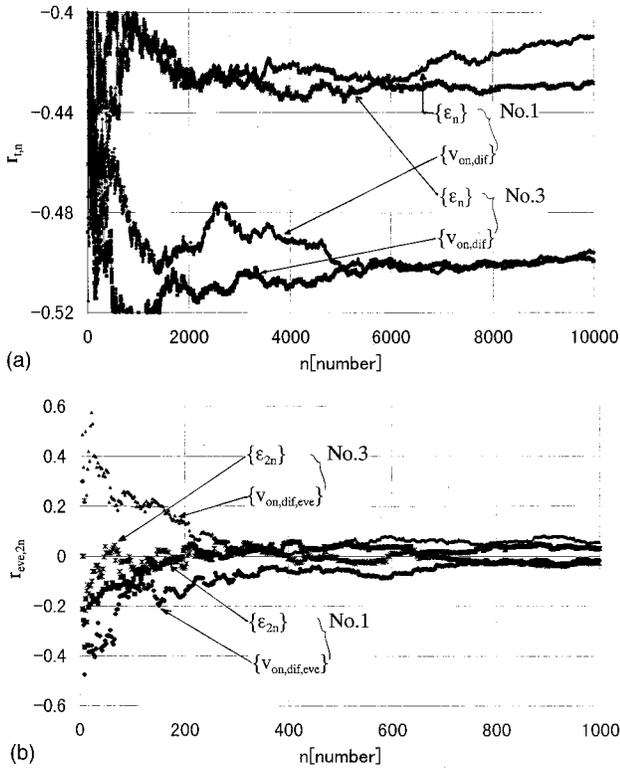


FIG. 11. Correlation in $\{\epsilon\}$ and $\{\epsilon_{2n}\}$. (a) On $\{\epsilon\}_{10\,000}^2$. (b) On $\{\epsilon_{2n}\}_{10\,000}^2$.

5. Degree of correlation between adjacent terms in $\{\epsilon\}$

For investigating the degree of the correlation in both $\{\epsilon\}_{10\,000}^2$ and $\{\epsilon_{2n}\}_{10\,000}^2$, we assume the following well-known relationship on the set of residual quantities:

$$\epsilon_n = \rho_1 \epsilon_{n-1} + \delta_n, \quad (22)$$

$$\epsilon_{2n} = \rho_{eve} \epsilon_{2(n-1)} + \delta_{2n} \quad (23)$$

and estimate these ρ_1 and ρ_{eve} as $r_{t,n}$ and $r_{eve,n}$ given by

$$r_{t,n} = \frac{\sum_{k=2}^n \epsilon_k \epsilon_{k-1}}{\sum_{k=2}^n \epsilon_k^2}, \quad (24)$$

$$r_{eve,2n} = \frac{\sum_{k=2}^{n/2} \epsilon_{2k} \epsilon_{2(k-1)}}{\sum_{k=2}^{n/2} \epsilon_{2k}^2}. \quad (25)$$

$r_{t,n}$ and $r_{eve,2n}$ are plotted in Figs. 11(a) and 11(b) as a function of n together with $\{v_{on,dif}\}$ and $\{v_{on,dif,2n}\}$, for consistency. $\{v_{on,dif,2n}\}$ is the subset comprised of even terms in $\{v_{on,dif}\}$, corresponding to $\{\epsilon_{2n}\}$. As to the increase of n , $r_{t,n}$ converges to

$$\begin{aligned} n \rightarrow 10\,000 &\Rightarrow r_{t,n} \\ &\rightarrow -0.409 \text{ to No. 1 and } -0.428 \text{ to No. 3} \end{aligned}$$

and r_{eve} does to

$$\begin{aligned} 2n \rightarrow 10\,000 &\Rightarrow r_{eve,2n} \\ &\rightarrow 0.0113 \text{ to No. 1 and } -0.0055 \text{ to No. 3.} \end{aligned}$$

In Fig. 11(b), $r_{eve,2n}$ in the region up to $n=1000$ is emphasized. When $r_{t,n}$ and $r_{eve,2n}$ are tested with a quantity u_n defined by

TABLE V. Correlation coefficients in $\{v_{on}\}$, $\{\epsilon\}$, $\{\epsilon_{2n}\}$, and $\{v_{on} - v_{on,av}\}$.

		r_n	u_n
No. 1	$\{v_{on}\}$ (30 V)	0.00249	0.2492
	$\{v_{on}\}$	0.3551	37.125
	$\{v_{on} - v_{on,av}\}$	0.01375	1.3749
	$\{v_{on,eve}\}$	0.3494	36.471
	$\{v_{on,eve} - v_{on,eve,av}\}$	0.01863	1.8629
	$\{v_{on,dif}\}$	-0.4958	-54.364
	$\{v_{on,dif,eve}\}$	0.01497	1.0583
	$\{\epsilon_n\}$	-0.4095	-43.490
	$\{\epsilon_{2n}\}$	0.01127	0.7968
	$\{v_{on}\}$ (30 V)	0.02386	2.3864
No. 3	$\{v_{on}\}$	0.7700	102.02
	$\{v_{on} - v_{on,av}\}$	0.02313	2.3128
	$\{v_{on,eve}\}$	0.7756	103.41
	$\{v_{on,eve} - v_{on,eve,av}\}$	0.01499	1.4992
	$\{v_{on,dif}\}$	-0.4994	-54.840
	$\{v_{on,dif,eve}\}$	0.00793	0.5606
	$\{\epsilon_n\}$	-0.4280	-45.735
	$\{\epsilon_{2n}\}$	0.00554	0.3916

$$u_n = \sqrt{(n-3)} \cdot z_n,$$

[where $z_n = \tanh^{-1} r_n$ in Eqs. (24) or (25)], (26)

which approximately obeys the normal distribution $N(0,1)$, the results summarized in Table V show $u = -43$ to -46 for $\{\epsilon\}$ and $u = 0.8$ to -0.4 for $\{\epsilon_{2n}\}$: (i) at the significance level 5% to both DMMs No. 1 and No. 3, the null hypothesis in $\{\epsilon\}$ is rejected for almost all n , $r_{t,n}$ being negative, which can be regarded as equivalent that the probability that the product of adjacent terms is negative is very high. As was indicated above, this reflects well that the number of run of length 1 is of absolutely many and (ii) in $\{\epsilon_{2n}\}$ it is acceptable for $n >$ at least 200. If Eq. (22) can be assumed in $\{\epsilon\}$, the correlation coefficient between ϵ_n and ϵ_{n-2} , i.e., ρ_{eve} , is given by

$$\rho_{eve} = \rho_1^2, \quad (27)$$

that is, the probability that $r_{eve,n} = r_{t,n}^2$ holds is high but in practice not so. The uncorrelation is directly connected to the high randomness in $\{\epsilon_{2n}\}$. That is, in $\{\epsilon\}$ the assumption of Eqs. (22) and (23) is not appropriate. $\{\epsilon_{2n}\}$ and $\{\epsilon\}$ are two different sets in a statistical sense. Error theory is at least applicable to $\{\epsilon_{2n}\}$. On the other hand, $\{v_{on}\}$ or $\{v_B\}$ have different properties from $\{\epsilon_{2n}\}$. The correlation coefficients $r_{t,n}$ calculated by

$$r_{t,n} = \frac{\sum_{k=2}^n v_{on,k} v_{on,k-1}}{\sum_{k=1}^n v_{on,k}^2} \quad (28)$$

are summarized in Table V together with others. Equation (28) gives $r_{t,n} = 0.3551$ for No. 1 and 0.7701 for No. 3. However, when $v_{on,k}$ in Eq. (28) is replaced by $v_{on,k} - v_{on,av}$ and $v_{on,2k}$ in

$$r_{eve,n} = \frac{\sum_{k=2}^{n/2} v_{on,2k} v_{on,2(k-1)}}{\sum_{k=1}^{n/2} v_{on,2k}^2} \quad (29)$$

is done by $v_{on,2k} - v_{on,eve,av}$ ($v_{on,eve,av}$ represents the average value of $\{v_{on,2n}\}$), both $r_{t,n}$ and $r_{eve,n}$ make us accept the null hypothesis, the uncorrelation.

$\{v_{on}\}_{(30\text{ V})}$ has a property that $r_{t,n}$ itself in Eq. (28) becomes almost equal to zero, the uncorrelation ($r_{eve,n} \cong 0$ holds.), being both uncorrelated and orthogonal (This means that $v_{on,av} = 0$ holds.), where the model of Eqs. (22) and (23) approximately holds. Furthermore, what arouses our interest in is the results shown in Fig. 11. Correlation in both $\{\epsilon\}_{10\,000}^2$ and $\{\epsilon_{2n}\}_{10\,000}^2$ is dominated by that of $\{v_{on,dif}\}$ and $\{v_{on,dif,eve}\}$, respectively.

IV. SUITABILITY OF $\{v_{on}\}$ AND $\{\epsilon_{2n}\}$ AS ZERO STATE MEASUREMENT

Through the above discussions, we found that $\{\epsilon_{2n}\}$ or $\{\epsilon_{2n-1}\}$ has a property much more suitable for the purpose of zero state measurement than $\{v_{on}\}$. However, either of them is a sequence of single measurement and one must make sure whether or not the obtained sequence is statistically under control, in a stable state. The stability of measurements which is called, in other words, ‘‘homogeneity’’ is investigated focusing our attention on subgroups obtained by properly partitioning $\{v_{on}\}$ or $\{\epsilon_{2n}\}$.

A. Homogeneity of the time series and histogram in some parts of $\{v_{on}\}$

The problem of how to partition $\{v_{on}\}$ or $\{\epsilon_{2n}\}$ into subgroups for monitoring their homogeneity remains for us. It is most recommended to have of the order of ten measurements in a group and to have several such groups.¹⁴ However, such measurement number is too small to play a role of representing any clear best estimate for $\{v_{on}\}$. In order to check this by picking up $\{v_{on}\}$ for DMM No. 1, the case of 20 measurements per group and totally 51 groups are treated. That is, $\{v_{on}\}$ is partitioned into subgroups $\{v_{on}\}_{20}^1, \{v_{on}\}_{40}^2, \{v_{on}\}_{60}^{41}, \dots, \{v_{on}\}_{1020}^{1001}$. These groups have the following properties on mean values and variances; the mean value which each group takes scatters in the range from 2.0×10^{-7} to 4.4×10^{-7} and the variance spreads from 6.003×10^{-14} to 2.54×10^{-13} . Under the assumption that $\{v_{on}\}_{10\,000}$ is a normal population with the variance $\sigma^2 = 1.41 \times 10^{-13}$ (from Table I), whether or not these variances can become the unbiased estimate of σ^2 is checked by applying χ^2 test ($\chi^2 = n \text{Var}/\sigma^2$) to them as follows; $\chi^2 = (20 \times 6.003 \times 10^{-14}) / (1.41 \times 10^{-13}) = 8.515$ and $\chi^2 = (20 \times 2.45 \times 10^{-13}) / (1.41 \times 10^{-13}) = 34.75$. These values are out of the significance limits $\chi_1^2 = 9.59$ and $\chi_2^2 = 34.17$ for the significance level $\alpha = 5\%$. The corresponding variance limits are 6.76×10^{-14} and 2.401×10^{-13} , respectively, and the group number, which remains within the limits, is 47, 92% of the total. The tendency like this is not so variant even if the number of groups is increased. About 8% of the total at least cannot be regarded as a sample from the same population $\{v_{on}\}$, none the less they belong to it. Such groups that are out of the significance limits are another population, statistically. In practice, an F -test on the admissibility of differences of variances indicates this. For these two groups having the variance of 6.003×10^{-14} and 2.45×10^{-13} , respectively, $F_0 = (2.45 \times 10^{-13}) / (6.003 \times 10^{-14}) = 4.23$ is larger than the boundary value 2.5 for the significance level $\alpha = 5\%$.

Next, as a case of 50 measurements per group let’s pick

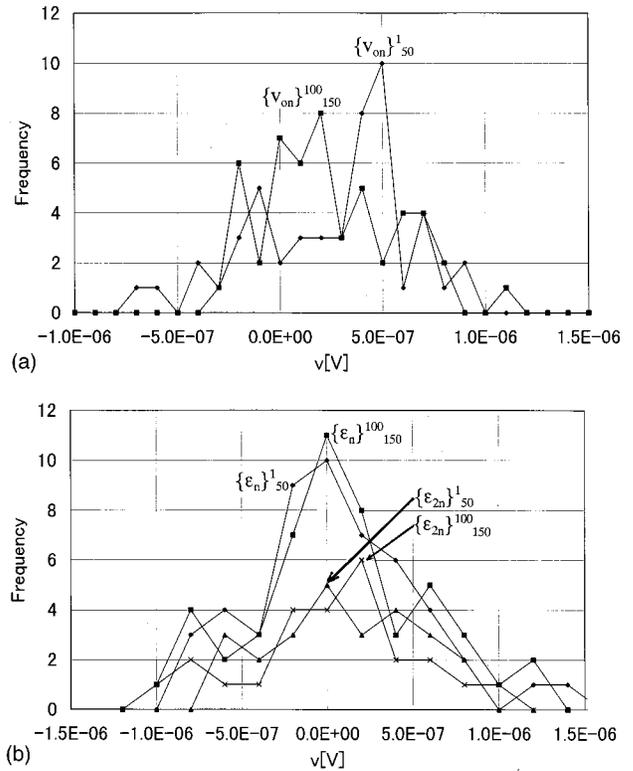


FIG. 12. Histograms for subgroups partitioned from $\{v_{on}\}$ and $\{\epsilon_n\}$ or $\{\epsilon_{2n}\}$ in No. 1. (a) For $\{v_{on}\}_{50}^1$ and $\{v_{on}\}_{150}^{100}$. (b) For $\{\epsilon_n\}_{50}^1$ and $\{\epsilon_n\}_{150}^{100}$, and $\{\epsilon_{2n}\}_{50}^1$ and $\{\epsilon_{2n}\}_{150}^{100}$.

two partitioned groups $\{v_{on}\}_{50}^1$ and $\{v_{on}\}_{150}^{101}$ of $\{v_{on}\}$ for DMM No. 1. Their histograms are shown in Fig. 12(a). These two at a glance clearly differ from each other. Their variances are 1.446×10^{-13} for $\{v_{on}\}_{50}^1$ and 1.061×10^{-13} for $\{v_{on}\}_{150}^{101}$. Similarly, χ^2 test ($\chi^2 = n \text{Var}/\sigma^2$) gives; $\chi^2 = (50 \times 1.446) / 1.41 = 51.28$ for $\{v_{on}\}_{50}^1$ and $\chi^2 = (50 \times 1.061) / 1.446 = 37.62$ for $\{v_{on}\}_{150}^{101}$, both of which remain within the significance points $\chi_1^2 = 32.4 \leq \chi^2 \leq \chi_2^2 = 71.4$ for the significance level $\alpha = 5\%$ and the null hypothesis that these two are samples produced from the same population $\{v_{on}\}$ becomes acceptable. Similarly under the same assumption, an F -test on the admissibility of differences between these two is: $F_0 = 1.446 / 1.061 = 1.36 < 1.60$ and $1 / 1.36 > 0.56$ at the significance level $\alpha = 5\%$ and the null hypothesis is acceptable. These suggest: For achieving high precision measurement of zero state based on $\{v_{on}\}$, a data number more than at least 50 has to be included in one measurement, though the zero voltage offset remains without becoming equal to zero. Even if several units of DMMs are utilized in parallel, we can not necessarily expect high accuracy and precision based on AZ-ON. Here, a comment which we would like to add to is that these are all based on the assumption that $\{v_{on}\}$ belongs to a normal distribution. Almost the same thing occurs for DMM No. 3. As mentioned before, we took 120 or 240 measurements per group for identifying the population $\{v_B\}$ of $\{v_{on}\}$ and $\{v_{on}\}_{(30\text{ V})}$. The background is based on the result through the above discussion.

B. Homogeneity of the time series and histogram in some parts of $\{\epsilon_n\}$ or $\{\epsilon_{2n}\}$

Let's consider the same thing for $\{\epsilon_{2n}\}$ and take up such a case that one group contains only ten observations; each of $\{\epsilon_{2k}, k=1-10\}$, $\{\epsilon_{2k}, k=11-20\}$, $\{\epsilon_{2k}, k=21-30\}$,..., giving a measurement group, and totally 51 groups are chosen. The mean value remains within $\pm 2.0 \times 10^{-7}$. The variance spreads from 3.667×10^{-14} to 4.467×10^{-13} . Their χ^2 test to $\{\epsilon_k\}_{10\,000}^2$ or $\{\epsilon_{2k}\}_{10\,000}^2$ gives $10 \times 3.667 \times 10^{-14} / 1.688 \times 10^{-13} = 2.17$ and $10 \times 4.467 \times 10^{-13} / 1.688 \times 10^{-13} = 26.46$. These values certainly are out of the significance points $\chi_1^2 = 3.25$ and $\chi_2^2 = 20.5$ for $\alpha = 5\%$. The groups which remain within the boundaries of 5.29×10^{-14} and 3.481×10^{-13} are 41 and occupy 81% of the total. This rate rises rapidly by increasing the measurement number per group.

Next, let's examine $\{\epsilon\}_{51}^2$ and $\{\epsilon\}_{151}^{101}$ and $\{\epsilon_{2n}\}_{51}^2$ and $\{\epsilon_{2n}\}_{151}^{101}$. Their histograms shown in Fig. 12(b) give an impression to us quite different from those in Fig. 12(a). They are stable, statistically under control. The histogram of $\{\epsilon\}_{151}^{101}$ almost coincides with that of $\{\epsilon\}_{51}^2$ and has a good symmetry with respect to zero. The same is also recognized in both $\{\epsilon_{2n}\}_{51}^2$ and $\{\epsilon_{2n}\}_{151}^{101}$. These two includes only 25 measurements in either group, nevertheless their variances can become an unbiased estimate of the variance of $\{\epsilon\}$. The null hypothesis on the variance between $\{\epsilon_{2n}\}_{51}^2$ and $\{\epsilon_{2n}\}_{151}^{101}$ is of course acceptable. In addition, their histograms are more satisfactory for symmetry around zero compared with that of $\{v_{on}\}_{50}^1$ or $\{v_{on}\}_{150}^{101}$. The distortion factor is 0.29 for $\{\epsilon\}_{51}^2$, 0.14 for $\{\epsilon\}_{151}^{101}$, and -0.25 for $\{\epsilon_{2n}\}_{51}^2$, 0.32 for $\{\epsilon_{2n}\}_{151}^{101}$, respectively, while -0.55 for $\{v_{on}\}_{50}^1$ and 0.48 for $\{v_{on}\}_{150}^{101}$. On $\{\epsilon_{2n}\}$ both $\{\epsilon_{2n}\}_{51}^2$ and $\{\epsilon_{2n}\}_{151}^{101}$ keep relatively well the shape of these $\{\epsilon\}_{51}^2$ and $\{\epsilon\}_{151}^{101}$ with the distortion factor of -0.25 and 0.32, respectively. These also indicate that $\{\epsilon_{2n}\}$ is more homogeneous than $\{v_{on}\}$ is. Why the histogram shown in Fig. 12(a) is forced into a deformed shape is due to a fact that $\{v_{on}\}$ is essentially inhomogeneous from looking at it partially. The preamplifier seems to have an influence on it. (The details will be presented in near future.)

Through these discussions, one important conclusion derived is that, since $\{v_A\}$ is considered to be largely influenced by $\{v_B\}$, the state of AZ-OFF has a more extensive potential of performing to their full specifications for measurement of μV level dc voltage than that of AZ-ON. A preferable procedure, as is suggested from the above discussions, is given at AZ-OFF as composed of two modes per measurement: v_s is first measured as $v'_{(1)}$, and subsequently for zero correction $v'_{(2)}$ is done under the short-circuited input terminals (see Fig. 1). Then, the corrected value v_m produced as $v_m = v'_{(1)} - v'_{(2)}$ is $v_m = v_s + v_{\text{error}}$, where v_{error} represents just the term of ϵ_{2n} . Thus, $\{\epsilon_{2n}\}$ is a set of error voltage. The evaluation is summarized as follows. Under a repetition measurement in which one repetition includes only ten observations, the mean value of the error voltage in each repetition remains within $\pm 2 \times 10^{-7}$ V and the s.d. specifying the measurement precision is 1.26×10^{-7} V. This gives a satisfactory performance for measuring μV level dc voltage.

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